

Financial Mathematics I

Exercise Sheet 0

This exercise sheet should not be submitted! We will discuss the solutions together during the tutorials on April 30, 2015.

Exercise 1 (Markowitz)

We consider the one-period Markowitz model with two assets P_1 and P_2 and corresponding relative returns R_1 and R_2 . We let

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0.9 \end{pmatrix} \quad \sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} = \begin{pmatrix} 0.1 & -0.1 \\ -0.1 & 0.15 \end{pmatrix}$$

denote the vector of expected returns and the covariance matrix of the assets, respectively. Solve the following optimization problem:

$$\text{Var}[\pi_1 R_1 + \pi_2 R_2] \longrightarrow \min !$$

subject to

$$\pi_1, \pi_2 \geq 0, \quad \pi_1 + \pi_2 = 1, \quad \mathbb{E}[\pi_1 R_1 + \pi_2 R_2] \geq 0.96.$$

Exercise 2 (Replication)

By building a portfolio of suitable European call and put options, construct an option with a final payoff that

- (i) leads to a gain if the underlying stock price does not move (significantly) between now and maturity.
- (ii) leads to a gain if the underlying stock price at maturity differs by a large amount from today's stock price.
- (iii) leads to an increasing gain if the stock price at maturity is slightly higher than today's price and to a constant gain if it is much higher than today's price.

What view about the market is reflected in each of the following strategies?

- (iv) *Bearish spread*: Buy one European call and sell a second one with the same expiry date, but a smaller strike price.
- (v) *Strangle*: Buy one European put and buy one European call with the same expiry date, but a larger strike price.