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Financial Mathematics I

Exercise Sheet 4

Please submit your solution by 17:00 on Monday, June 01.

Exercise 1 (Hitting Times)
Let $\{(X_t, \mathcal{F}_t)\}_{t\geq 0}$ be a right-continuous $\mathbb{R}$-valued stochastic process and assume that $\{\mathcal{F}_t\}_{t\geq 0}$ satisfies the usual conditions. Let furthermore $a \in \mathbb{R}$ be given. Prove or disprove the following statements:

(i) The random variable $T(\omega) := \inf\{t \geq 0 : X_t(\omega) \geq a\}$ is a stopping time.

(ii) The random variable $T(\omega) := \sup\{t \geq 0 : X_t(\omega) \geq a\}$ is a stopping time.

Exercise 2 (Exponential Martingales)
Let W be a one-dimensional Brownian motion and let $Y \in H^2[0, T]$. Define a process Z through

$$Z_t := \exp\left(-\int_0^t Y_s\,dW_s - \frac{1}{2}\int_0^t Y_s^2\,ds\right), \qquad t \in [0, T].$$

Show that

$$Z_t = 1 - \int_0^t Z_s Y_s\,dW_s, \qquad t \in [0, T],$$

and conclude that Z is a martingale if $ZY \in L^2[0, T]$.