CHARACTER THEORY OF FINITE GROUPS SS 2022
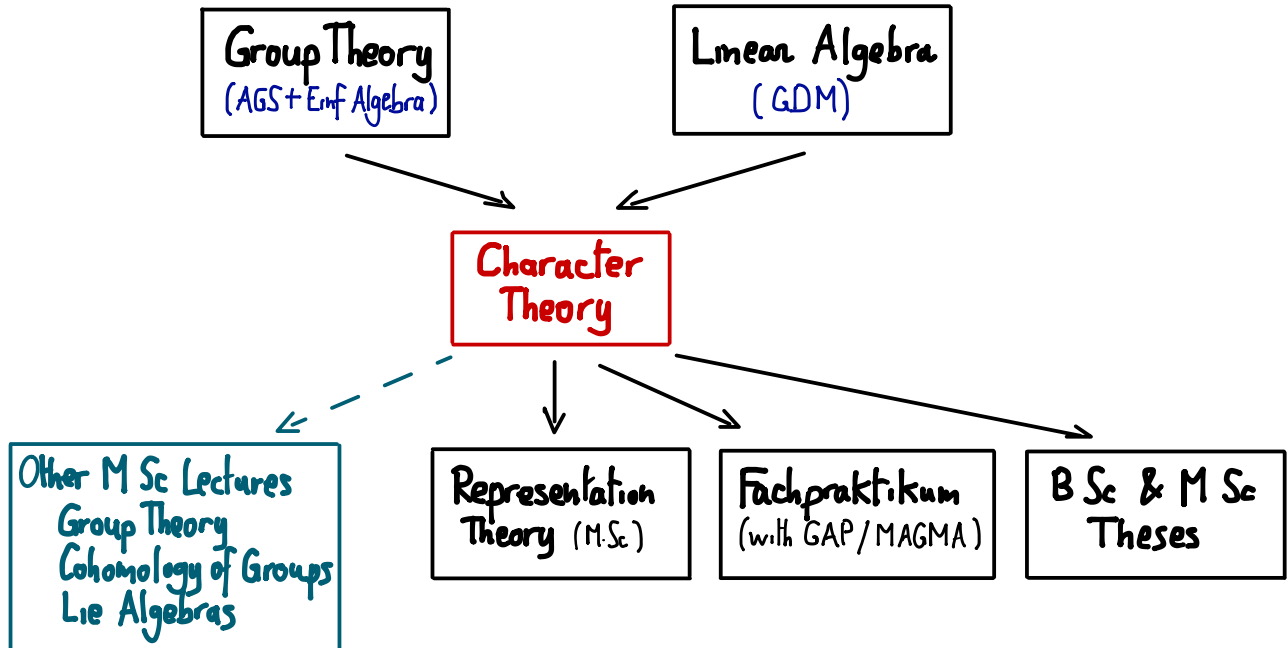
A ORGANISATION

- **LECTURE** $\longrightarrow$ Mondays 10 00 - 11 30 (13×)

- **EXERCISES** $\longrightarrow$ Fridays 12 00 - 13 30 (7×)   Start 1st Week

  $\longrightarrow$ Assisstant Bernhard Böhmler

  $\longrightarrow$ Exercise Sheets   to be downloaded from my webpage.

SCHEINE To obtain an Übungsschein you need – 2 out of 8 points on the exercises to hand in (Sheets 2 to 7)

 – attendance to the exercise class: in part.: present 2 solutions at the BB.

CHARACTER THEORY OF FINITE GROUPS SS 2022

B BEFORE AND AFTER

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        ┌─────────────────┐        ┌─────────────────┐
        │ Group Theory    │        │ Linear Algebra  │
        │ (AGS + Einf Algebra) │   │ ( GDM)          │
        └─────────────────┘        └─────────────────┘
                 │                           │
                 ↘                         ↙
                    ┌──────────────┐
                    │  Character   │
                    │  Theory      │
                    └──────────────┘
          ⟍           │        ↘            ↘
       ↙           ↓          ↘               ↘
┌──────────────────┐  ┌──────────────┐ ┌──────────────┐ ┌──────────────┐
│ Other M Sc Lectures│ │ Representation│ │ Fachpraktikum│ │ B Sc & M Sc  │
│  Group Theory    │  │ Theory (M.Sc)│ │(with GAP/MAGMA)│ │  Theses     │
│  Cohomology of Groups│ └──────────────┘ └──────────────┘ └──────────────┘
│  Lie Algebras    │
└──────────────────┘
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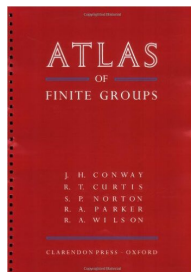
C   INTRODUCTION

(1) Character Theory in the 2020 Mathematics Subject Classification

www.ams.org/msc/msc2020.pdf

(2) The ATLAS OF FINITE GROUPS

J.H. CONWAY, R.T. CURTIS, S.P. NORTON, R. PARKER, R.A. WILSON, *Atlas of Finite Groups.* Clarendon Press, Oxford, 1985.

$$\boxed{\text{CHARACTER THEORY OF}}$$
$$\boxed{\text{FINITE GROUPS SS 2022}}$$

C INTRODUCTION

(3) Aims of the lecture

$\longrightarrow$ Character Tables of finite groups

       − What are they?
       − Elementary methods to compute them
       − What information about finite groups
         do they contain?

$\longrightarrow$ Give a proof of

Burnside's $p^a q^b$ - Theorem   Let p, q be prime numbers and let $a, b \in \mathbb{N}_0$
             (1911)   Then any finite group of order $p^a q^b$ is soluble

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Example of a character table

	[1]	[q_2]	[q_3]	[q_4]	[q_5]	[q_6]
χ_1	1	1	1	1	1	1
χ_2	3	-1	1	0	α	$\bar{\alpha}$
χ_3	3	-1	1	0	$\bar{\alpha}$	α
χ_4	6	2	0	0	-1	-1
χ_5	7	-1	-1	1	0	0
χ_6	8	0	0	-1	1	1

$$\alpha := \frac{-1 + i\sqrt{7}}{2}$$

Conclusions (E.g.)

* $|G| = 168$
* G non abelian
* $Z(G) = \{1\}$
* G simple

...

$$\left(\Rightarrow G \cong PSL_2(\mathbb{F}_7) \right)$$

Character Theory of
finite Groups SS 2022

D PROGRAMME

I Linear representations of finite groups

II Characters of representations

III The character table

IV Burnside's $p^a q^b$-Theorem

V Induction and restriction

VI Brauer's Characterization of characters