

This text constitutes a faithful transcript of the lecture **Character Theory of Finite Groups** held at the TU Kaiserslautern during the Summer Semester 2022 (14 Weeks, 2SWS Lecture + 1SWS Exercises).

Together with the necessary theoretical foundations the main aims of this lecture are to:

- provide students with a modern approach to **finite group theory**;
- learn about the **ordinary character theory of finite groups**;
- learn about the **applications** of the latter theory to **finite group theory**, such as for example the proof of Burnside's $p^a q^b$ -Theorem.

We assume as pre-requisites bachelor-level algebra courses dealing with *linear algebra* and *elementary group theory*, such as the standard lectures *Grundlagen der Mathematik*, *Algebraische Strukturen*, and *Einführung in die Algebra*.

The exercises mentioned in the text are important for the development of the lecture and the general understanding of the topics. Further exercises can be found in the fortnightly exercise sheets.

The content of the appendix is important for the lecture. Some of you may have encountered (or will encounter) this material in other algebra lectures. For this reason, no direct question on the appendix will be asked in the oral exam.

Books and lecture notes which were used to prepare these lecture notes are the following.

Textbooks:

- [Dor71] L. Dornhoff. *Group representation theory. Part A: Ordinary representation theory*. Marcel Dekker, Inc., New York, 1971.
- [Hup98] B. Huppert. *Character theory of finite groups*. Vol. 25. Walter de Gruyter & Co., Berlin, 1998.
- [Isa94] I. M. Isaacs. *Character theory of finite groups*. Dover Publications, Inc., New York, 1994.
- [JL01] G. James and M. Liebeck. *Representations and characters of groups*. Second. Cambridge University Press, New York, 2001.
- [LP10] K. Lux and H. Pahlings. *Representations of groups*. Vol. 124. Cambridge University Press, Cambridge, 2010.

- [NT89] H. Nagao and Y. Tsushima. *Representations of finite groups*. Academic Press, Inc., Boston, MA, 1989.
- [Rot10] J. J. Rotman. *Advanced modern algebra. 2nd ed.* Providence, RI: American Mathematical Society (AMS), 2010.
- [Ser77] J.-P. Serre. *Linear representations of finite groups*. Springer-Verlag, New York-Heidelberg, 1977.
- [Ser78] J.-P. Serre. *Représentations linéaires des groupes finis*. revised. Hermann, Paris, 1978.
- [Web16] P. Webb. *A course in finite group representation theory*. Vol. 161. Cambridge University Press, Cambridge, 2016.

Lecture Notes:

- [Gec14] M. Geck. *Algebra: Gruppen, Ringe, Körper (mit einer Einführung in die Darstellungstheorie endlicher Gruppen)*. Edition Delkhofen, 2014.
- [Kül13] B. Külshammer. *Darstellungstheorie*. 2013. URL: <http://www.minet.uni-jena.de/algebra/skripten/dt/dt-2010/dt.pdf>.
- [Mal16] G. Malle. *Characters of finite groups*. Lecture Notes SS 2016, TU Kaiserslautern. 2016.
- [Thé05] J. Thévenaz. *Représentations linéaires des groupes finis*. Lecture Notes WS 2004/05, EPFL. 2005.

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