

Übungen zu Lie-Algebren — Blatt 1

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Abgabetermin: **Di, 25.10.11, 11:30 Uhr**

PD Dr. M. Cuntz
WS 11/12

Let K be a field.

Exercise 1: Let L be a Lie algebra over K and define $\text{ad} : L \rightarrow \mathfrak{gl}(L)$ by $\text{ad}(x)(y) := [x, y]$ for $x, y \in L$.

- (a) Show that ad is a linear map.
- (b) Show that $\text{ad}(x)$ is a linear map for all $x \in L$.
- (c) Show that ad is a Lie algebra homomorphism.
- (d) Determine its kernel $\ker(\text{ad})$.

Exercise 2: Let $\varphi : L_1 \rightarrow L_2$ be a homomorphism of Lie algebras. Show that $\ker(\varphi)$ is an ideal of L_1 and the image $\text{im}(\varphi)$ is a Lie subalgebra of L_2 .

Exercise 3: Let $\mathfrak{sp}_{2n}(K)$ be the symplectic Lie algebra on the $2n$ -dimensional K -vector space K^{2n} .

- (a) Show that $\mathfrak{sp}_{2n}(K) \leq \mathfrak{sl}_{2n}(K)$.
- (b) Determine a basis of $\mathfrak{sp}_{2n}(K)$ and compute the dimension $\dim(\mathfrak{sp}_{2n}(K))$.
- (c) Same for the orthogonal Lie algebra $\mathfrak{so}_{2n}(K)$.