

Übungen zu Lie-Algebren — Blatt 9

Prof. Dr. G. Malle

Abgabetermin: **Di, 20.12.11, 11:30 Uhr**

PD Dr. M. Cuntz

WS 11/12

Exercise 31: Let α, β be roots in a root system Φ and assume that $\alpha - \beta \in \Phi$. Does it follow that $(\alpha, \beta) > 0$?

Exercise 32: Let $P, Q \in \mathbb{Z}^{n \times n}$ have non-negative integral entries. Show that if $PQ = I_n$ then P and Q are permutation matrices, i.e., each line and each column contains one entry 1 and 0 otherwise.

Exercise 33: Prove that the Weyl group of a root system is isomorphic to the direct product of the Weyl groups of its irreducible components.

Exercise 34: Suppose that R is a root system in the euclidean vector space E . Show that

$$\check{R} := \left\{ \frac{2\alpha}{(\alpha, \alpha)} \mid \alpha \in R \right\}$$

is also a root system in E . Show that the Cartan matrix of \check{R} is the transpose of the Cartan matrix of R , and that R, \check{R} have isomorphic Weyl groups. \check{R} is called the *dual root system* to R .