In this exercise sheet, K denotes an algebraically closed field of characteristic zero, and all Lie algebras are assumed to be finite-dimensional.

Exercise 17.
Let $L_1, \dots, L_t$ be simple ideals of a Lie algebra L satisfying $L = L_1 \oplus \cdots \oplus L_t$. Show that L is semisimple.

Exercise 18.
Let V and W be two L-modules. Then $\mathrm{Hom}_K(V, W)$ is an L-module via

$$(x \cdot f)(v) := xf(v) - f(xv),\ x \in L,\ f \in \mathrm{Hom}_K(V, W),\ v \in V.$$

Exercise 19.
Let L be a 3-dimensional Lie algebra over K, with basis $\{x, y, z\}$, and Lie brackets $[x, y] = z$, $[x, z] = y, [y, z] = 0$. (Check that this defines a Lie algebra structure.) Compute the following:

1. $Z(L)$.
2. $[L, L]$.
3. The radical S_κ of the Killing form κ on L.
4. A matrix representation associated with the adjoint representation of L.

Exercise 20.
Assume K is an algebraically closed field of characteristic zero and let L be a soluble Lie algebra over K. Prove that all the irreducible representations of L are 1-dimensional.