Solution 6

Looking from a point on the z-axis we see

\[ y^2 + z^2 = 1 \]
\[ x^2 + z^2 = 1 \]

Note that the projection to the x-y-plane of the intersection curves of \( x^2 + z^2 = 1 \) and \( y^2 + z^2 = 1 \) are two diagonal lines:

\[ x^2 + z^2 = 1 = y^2 + z^2 \Rightarrow 0 = x^2 - y^2 = (x-y)(x+y) \]

\( \Rightarrow \) \( y = x \) or \( y = -x \).

The volume is

\[ V = 8 \int_{-\pi/4}^{\pi/4} \int_{-\pi/4}^{\pi/4} \int_0^1 \, dz \, dr \, d\theta \]

\[ = 8 \int_{-\pi/4}^{\pi/4} \int_{-\pi/4}^{\pi/4} \left[ \frac{1}{2} \cos^2 \theta - \frac{1}{2} \cos^2 \theta \right] \, d\theta \]

\[ = \frac{8}{3} \int_{-\pi/4}^{\pi/4} \frac{\sin^3 \theta}{\cos^2 \theta} \, d\theta \]

\[ = \frac{8}{3} \left[ \tan \theta \right]_{-\pi/4}^{\pi/4} = \frac{8}{3} \left( (1) - (1) \right) = \frac{8}{3} \]