3.2.1 By 3.2.3, \( f : (a, b) \rightarrow \mathbb{R} \) is continuous, so the MVT applies to any interval \([x_1, y] \subset (a, b)\) and gives an \( z \in (x_1, y) \) s.t. \( f(x_1) - f(y) = f'(z)(x_1 - y) = 0 \). Thus, \( f(x_1) = f(y) \) for all \( x_1, y \in (a, b) \) and \( f \) is constant.

3.2.3 Fix \( x_0 \in (a, b) \). Then

\[
\lim_{x \to x_0} (f(x) - f(x_0)) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \cdot \lim_{x \to x_0} (x - x_0) = f'(x_0) \cdot 0 = 0,
\]

and hence \( f \) is continuous at \( x_0 \).

3.2.5 Note that some of the zeros of \( f \) and \( f' \) may coincide. But if \( x \) is a zero of \( f \) and \( f' \), then it is also a zero of \( f' = f'p + f'q \).

Combining this observation w/ Cor. 4, we conclude that \( f' \) has at least 7 zeros. Applying Cor. 1 once more shows that \( f(2) \) has at least 6 zeros.

3.2.9 Let \( M \) be a bound for \( f' \). Then by 3.2.3 and the MVT, \( |f(a) - f(b)| \leq M |a - b| \). So \( f \) is Lipschitz-continuous and hence uniformly continuous. Indeed, for \( \epsilon > 0 \) we can pick \( \delta = \frac{\epsilon}{M} \). Then \( \forall a, b \) s.t. \( |a - b| < \delta \) we have \( |f(a) - f(b)| \leq M \delta = \epsilon \).