3.3.7 Let \( S \subseteq \mathbb{R}^n \) be compact with \( \delta = \delta_s \), and \( f, g \in C^0(S) \cap C^1(S) \) such that \( f|\delta_s = g|\delta_s \). Then \( Df(p_0) = Dg(p_0) \) for some \( p_0 \in S \).

**proof:** By subtracting \( g \) from both \( f \) and \( g \), we reduce the problem to the case where \( f = 0 \).

By Theorem 2.4.11, \( f \) takes a maximum and a minimum value on \( S \). If both equal zero, then \( f = 0 \) and any \( p_0 \in S \) works. Otherwise, \( f \) takes an extremum at some \( p_0 \in S \ \delta_s = \delta_s \), since \( f|\delta_s = 0 \). Then \( Df(p_0) = 0 \) by Theorem 3.3.11.