TECHNISCHE UNIVERSITÄT
KAISERSLAUTERN

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Fachbereich Mathematik

Algebraic Geometry

Summer Semester 2013 - Problem Set 10

Due June 28, 2013, 1:00 pm

Problem 1. Resolve all singularities of X by subsequent blowups:

(a) $X = Z(x^2 - x^4 - y^4) \subset \mathbb{A}^2_{\mathbb{C}}$,

(b) $X = Z(x^3 - y^5) \subset \mathbb{A}^2_{\mathbb{C}}$,

(c) $X = Z(y^2 - x^n) \subset \mathbb{A}^2_{\mathbb{C}}$, $n \in \mathbb{N}$.

Problem 2. Show that a general hypersurface in $\mathbb{P}^n_{\mathbb{C}}$ of degree d is smooth.

Problem 3. Find all closed points in $\operatorname{Spec}(\mathbb{R}[x, y])$ and their residue fields.

Problem 4. For the rings R below, consider $f = y^2 - x^2 - x^3 \in R$ and let $X = \operatorname{Spec} R/\langle f \rangle$. Describe X set-theoretically and decide whether it is irreducible.

(a) $R = \mathbb{C}[x, y]$

(b) $R = \mathbb{C}[x, y]_{\langle x,y \rangle}$

(c) $R = \mathbb{C}[\![x, y]\!]$