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Algebraic Geometry

Summer Semester 2013 - Problem Set 11

Due July 5, 2013, 1:00 pm

Exercise 1. Find an example of a scheme X subject to the given conditions or prove that such a scheme does not exist.

(a) $\#X = \infty$, $\dim X = 0$.

(b) $\#X = 1$, $\dim X = 1$.

(c) $\#X = 2$, $\dim X = 1$.

(d) $X = \operatorname{Spec} R$, $R \subseteq \mathbb{C}[x]$, $\dim X = 2$.

Exercise 2. Let X be a scheme of finite type over an algebraically closed field k. Show that the closed points of X are dense in X. Show by example that the hypotheses on X are necessary.

Exercise 3. Are the schemes $X = \operatorname{Spec}(\mathbb{C}[x, y, z]/\langle xy, xz, yz\rangle)$ and $Y = \operatorname{Spec}(\mathbb{C}[x, y]/\langle xy(x - y)\rangle)$ isomorphic? (Hint: Compare the local rings at the origin.)

Exercise 4. Let X be a scheme and $Y \subseteq X$ an irreducible closed subset with generic point η. Show that $\mathscr{O}_{X,Y} := \mathscr{O}_{X,\eta} = \{(U, \phi) \mid U \subseteq X \text{ open}, U \cap Y \neq \emptyset, \phi \in \mathscr{O}_X(U)\}/_\sim$ where $(U, \phi) \sim (U', \phi')$ if $\phi|_V = \phi'|_V$ for some $V \subseteq X$ open with $V \subseteq U \cap U'$. (In case X comes from a variety, $\mathscr{O}_{X,X} = K(X)$ is the function field of X.)