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Algebraic Geometry

Summer Semester 2013 - Problem Set 9

Due June 21, 2013, 1:00 pm

In all exercises, the ground field k is assumed to be algebraically closed.

Problem 1. For any two varieties X and Y the following conditions are equivalent:

(a) X and Y are birationally equivalent,

(b) there are open subsets $U \subset X$ and $V \subset Y$ with U isomorphic to V,

(c) $K(X) \cong K(Y)$ as k-algebras.

Problem 2. Any variety X of dimension r is birational to a hypersurface Y in $\mathbb{P}^{r+1}$.
(Hint: Use Problem 1 and the facts about separable/separably generated field extensions.)

Problem 3. Let Y be a variety. Then the set $\operatorname{Sing} Y$ of singular points of Y is a proper closed subset of Y. (Hint: Use Problem 2)