Irregular primes to two billion

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ANTS XII, Kaiserslautern
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We did $p < 2^{31} = 2,147,483,648$. 

Running time: 8.6 million core hours = 981 core years.

Results: nothing interesting happened.

Kummer–Vandiver conjecture holds for all $p$.

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