Modelling the local strut thickness of open foams based on 3D image data

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Random tessellations are widely used to model the microstructure of open foams. In this paper, we apply a two-step modelling approach: First, a tessellation model is fitted to the cell system of the foam. Subsequently, a model for the solid component is obtained by adaptable dilation of the tessellation’s edges. Geometric characteristics for the model fit are estimated from a 3D image of the foam. In particular, the locally varying strut thickness is analysed and described by a parametric model yielding the size map for the dilation. The approach is applied to an open cell copper foam and the model is validated using the densities of the intrinsic volumes.

Introduction

Open cell metal foams are versatile materials which are used in many application areas including heat exchangers, catalysts or sound absorbers. The physical properties of a foam are highly affected by its microstructure, for instance the porosity of the material, the size and shape of the foam’s cells or the geometry of its struts. An understanding of the change of the foam’s properties with altering microstructure is crucial for the optimisation of foams for specific applications. Foam models from stochastic geometry are powerful tools for investigating these relations.

Edge systems of random tessellations are often used as models for open foams. The models are fitted to real materials using geometric characteristics that are estimated from tomographic images of the foams. Realisations of foams with slightly modified microstructure can then be generated by changing the model parameters. Numerical simulations in these virtual foam samples allow for an investigation of relations between the geometric structure of a material and its physical properties.

The microstructure of an open foam is built from an interconnected network of struts with locally varying thickness. It can be modelled by the adaptively dilated edge system of a Laguerre tessellation [7]. To obtain the size map for the adaptive dilation, we introduce a skeletonisation based estimation of the local thickness of the individual strut segments. Based on this data we model the thickness profile by polynomials, that allow to reproduce the observed strut thickness profiles in the microstructure model.

Analysis of the CT data

The material under consideration is an open celled Duocell® copper foam of dimensions 250 mm × 25 mm × 25 mm. For microstructure analysis using μCT a cube with an edge length of 25 mm was cut from the rod, resulting in an image with 630³ voxels and voxel size of 38.15 µm. To fit an exact geometric model to the foam, knowledge of the empirical distributions of its cell characteristics, like volume or diameter, as well as a description of the varying strut thickness is crucial. For both we start with the binarisation of the volume image.

To estimate the cell characteristics we apply the cell reconstruction method described in [8]. Figure 1 depicts sectional images of the procedure. The characteristics of the foam cells are computed using the MAVI software package [3]. We use minus-sampling edge correction to avoid edge effects of cells hitting the image boundary. Furthermore, we found the cells slightly compressed along the x- and z-axis. An isotropic structure was obtained by scaling the cells by 1.28 along the x- and 1.23 along the z-axis.

Figure 1: Sectional images from the cell reconstruction method (from left to right): binarisation, Euclidean distance, its complement and reconstructed cell system

The procedure of characterising the locally varying strut thickness consists of two steps[6]. In the first step we compute the skeleton Y of the foam - a one pixel thick subset of its solid component X that is centred w.r.t. the Euclidean distance. In the second step the skeleton is decomposed into curve segments (struts) and curve junction (nodes).

Each strut of the foam contributes a curve segment Z in the skeleton Y. Let us denote the length of Z by ℓ. We define the local strut thickness at a point x of Z as the radius of the largest ball with centre x inscribed in X. Using the distance \( \xi = \xi(x) = \frac{\ell}{2} \) of x to the strut centre as a variable we obtain the thickness function \( \rho_\xi(\ell) \) of Z. The spherical contact profile \( P_\xi(\ell) \) of the entire foam is obtained as the mean thickness at distance \( \xi \) computed over all thickness profiles \( \rho(\ell) \) where the length of Z equals \( \ell \). To avoid edge effects, the struts contributing to \( P_\xi(\ell) \) are chosen using a minus-sampling edge correction.

Modelling the cell system

Based on the geometric characteristics of the foam estimated from the CT image we will now fit a stochastic model to the foam structure. In the first step of the modelling procedure we fit a random tessellation model to the cell system of the foam. We use a random Laguerre tessellation, which is a generalisation of the well-known Voronoi tessellation [5]. It is defined as follows:
Denote by $s(x,r)$ a sphere in $\mathbb{R}^d$ with center $x$ and radius $r$ and let $S$ be a locally finite set of spheres, which means that each bounded subset of $\mathbb{R}^d$ is intersected only by a finite number of spheres. The Laguerre cell $C(s(x,r), S)$ generated by a ball $s(x,r)$ consists of all points $y \in \mathbb{R}^d$ which are closest to $s(x,r)$ with respect to the so-called power distance $\text{pow}(y, s(x,r)) = ||x-y||^2 - r^2$, where $|| \cdot ||$ denotes the Euclidean distance. That means

$$C(s(x,r), S) = \{ y \in \mathbb{R}^d : \text{pow}(y, s(x,r)) \leq \text{pow}(y, s(x',r)) \text{ for all } s(x',r) \in S \}.$$ 

The Laguerre tessellation $L(S)$ is the set of Laguerre cells of balls contained in $S$. It is a space-filling system of convex polytopes. If all radii are equal, the Voronoi tessellation of the set of ball centres is obtained.

The foam cells are modelled by a Laguerre tessellation generated by a dense packing of non-overlapping balls which is simulated using the force-biased algorithm [1]. This model was chosen to reproduce the regular shape of the foam cells. A further advantage of this model is that each cell completely contains its generating ball. This allows a certain control over the volume distribution of the tessellation cells. Unfortunately, no analytic formulae relating the geometric characteristics of the tessellation cells with the parameters of the ball packing are available. To avoid a time-consuming model fit based on Monte-Carlo simulations we apply the fitting procedure introduced in [9] which is explained in detail in [10]. For this approach, a log-normal distribution is chosen for the volumes of the generating balls. The intensity of the ball packing equals the intensity of the foam cells which is estimated from the image as 18.25 per cm$^3$. The parameters of the model are the packing density $j$ and the coefficient of variation (CV) of the volume distribution of the ball packing. The deviation of the models from the foam sample is measured using the relative distance measure where the eight entries of $c = (c_1, \ldots, c_8)$ and $m = (m_1, \ldots, m_8)$ are the means and standard deviations of the volume $v$, surface area $s$, number of facets $f$, and diameter $d$ of the cells of the original foam and the model, respectively. In our case, the best-fit model is a Laguerre tessellation of balls with parameters $j = 80\%$ and $\text{CV} = 0.23$.

### Modelling the strut system

The solid component of the foam is modelled by adaptively dilating the edge system of the Laguerre tessellation by a ball [2].

To define the size map for the adaptive dilation given the ball radius at an edge point $x$ we apply the best two-parameter polynomial model found in [6], i.e.

$$P_{24}(\xi) = a_1 \xi^2 + a_2 \xi + 1 \quad \text{and} \quad P_{246}(\xi) = a_3 \xi^3 + a_4 \xi^2 + a_5 \xi + 1.$$ 

The models are defined scale free, thus assuming $P.(0) = 1$ and the length of the struts to be one. To achieve this, we normalise the data by mid-span thickness $P_x(0, \ell)$ and length $\ell$.

As the spherical contact profile $P_x(\xi, \ell)$ shows a dependence on length $\ell$ [6], we subdivided the data into its 10 length decile ranges $q_i = (x_i, x_{i+1})$, with $i = 0, \ldots, 9$, where $x_i$ denotes the $i$th decile of the strut length distribution. The coefficients of the two models $P_{24}(\xi)$ and $P_{246}(\xi)$ are then determined individually for each decile using weighted least squares. During the adaptive dilation the size map is chosen according to the edge length classes given by $q_i$. To obtain a smooth transition at the link of two struts we fixed the node size by scaling the models according $P_x(\xi) \times P_\ell$ with $P_\ell = n_\ell / P.(0.5)$, where $n_\ell$ denotes the mean radius at the nodes for the entire foam.

### Results

A visualisation of one of the models along with the image of the foam obtained by $\mu$CT is shown in Figure 1.

![Visualisations of the μCT image of the foam (left) and a realisation of the model using $P_{24}$ (right).](image)

The model is validated using the densities of the intrinsic volumes, i.e. the volume density $V$, the specific surface area $S_V$, and the densities of the integral of mean curvature $M_v$ and the Euler characteristic $\chi$. Table 1 summarises the average deviation of 25 realisations of each model from the values estimated from the data. Both models differ by less than three percent in $\chi$. As both structures are based on the same tessellation model this result is not surprising. $P_{24}$ has almost the same volume density as the data, while for $P_{246}$ it is about 10 % higher. This seems to be an effect of the scaling used to produce constant node sizes. The higher order polynomials of $P_{246}$ tend to flatten around $\xi = 0.5$ and thus produce slightly higher values for $P_\ell$. As the result both the volume density and the specific surface area of the generated structure increase.

While the specific surface area is fitted better by $P_{246}$, the deviation in $M_v$ is bigger than for $P_{24}$. One reason for this behaviour is that $P_{246}$ better fits the flat zones in the central parts of the struts and, thus, the profile of the resulting structure is less...
curved. Moreover, we did not include closed faces into the model that were present in the real foam. Hence, the \( M_V \) of the model is generally smaller than in the real foam.

Table 1: Mean deviation of the intrinsic volumes of 25 realisations of each model compared to the data

<table>
<thead>
<tr>
<th>Data</th>
<th>Dev. ( P_{24} ) (%)</th>
<th>Dev. ( P_{246} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_V )</td>
<td>0.1206</td>
<td>-2.53</td>
</tr>
<tr>
<td>( S_V ) (( \text{mm}^{-1} ))</td>
<td>6.786e5</td>
<td>7-27</td>
</tr>
<tr>
<td>( M_V ) (( \text{mm}^{-2} ))</td>
<td>8.474e11</td>
<td>-10.48</td>
</tr>
<tr>
<td>( \chi_V ) (( \text{mm}^{-3} ))</td>
<td>-1.390e17</td>
<td>+2.81</td>
</tr>
</tbody>
</table>

Figure 3 depicts the absolute differences of the volume density of the parallel sets of the structure model using \( P_{246} \) and the volume density of the data. That is, the absolute difference of the data’s \( V_V \) and the \( V_V \) estimated from the model structure dilated or eroded by a ball of radius \( r \), respectively. The minimum of the curve indicates the radius at which the optimal model is obtained. If this minimum is taken at \( r = 0 \), the model cannot be improved w.r.t. \( V_V \) by erosion or dilation and thus is optimal. The models based on \( P_{24} \) and \( P_{246} \) are optimal in this sense (the curves for both models have a similar shape).

![Figure 3: Comparison of the absolute difference of \( V_V \) in the data with \( V_V \) of the parallel sets of the model using \( P_{246} \). Negative radii correspond to erosion.](image)

**Discussion**

In [4] it was shown that realistic models for complex foam structures arise from combining image analysis with models from stochastic geometry and adaptable morphology. However, determining the size map for the adaptable dilation was tedious. In this paper we presented a procedure to derive the size map directly from the data.

We suggest a two step model fitting procedure: First a Laguerre Tessellation is fitted to the reconstructed cell system of the foam. In the second step we estimate the local thickness of the foam’s edge system and fit the two best polynomial model found in [6]. The edge system of the tessellation is then adaptively dilated using the polynomial model as size map.

The structure models that we obtain by this procedure are optimal w.r.t. to volume density. That is, it could not be improved by dilation or erosion. The deviations of the densities of the volume, the surface and Euler characteristic kept below 10 % for both models - without separately considering them as parameters for the model fit.

However, a deviation up to 16 % was observed for \( M_V \). This could be ascribed to closed faces in the real foam that were not reproduced in the structure model. Also, the surface of the real foam is slightly disturbed by noise. As noise has an impact on the estimation of \( M_V \), this leads to an overestimation for the real foam.
References