LAUTENSACK ET AL: 3d image analysis of foams using random tessellations

Claudia Lautensack
Fraunhofer Institut Techno- und Wirtschaftsmathematik (ITWM), Gottlieb-Daimler-Str. 49,
D-67663 Kaiserslautern
claudia.lautensack@itwm.fraunhofer.de

Tetyana Sych
Fraunhofer Institut Techno- und Wirtschaftsmathematik (ITWM), Gottlieb-Daimler-Str. 49,
D-67663 Kaiserslautern
tetyana.sych@itwm.fraunhofer.de

ABSTRACT

Volume image analysis provides a number of methods for the characterization of the microstructure of open foams. Mean values of characteristics of the edge system are measured directly from the volume image. Further characteristics like the intensity and mean size of the cells are obtained using model assumptions where the edge system of the foam is interpreted as a realization of a random closed set. Homogeneous random tessellations provide a suitable model for foam structures. However, their cells often lack the degree of regularity observed in real data. In this respect some deterministic models seem to be closer to realistic structures, but do not capture the microscopic heterogeneity of real foams. In this paper, the influence of the choice of the model assumptions on the obtained mean values is studied.

Keywords: foam structures, image analysis, random tessellations, volume images, Voronoi tessellation, Minkowski functionals.

INTRODUCTION

Foams (ceramic, polymer or metal) are new materials of high interest in a wide range of application areas. Due to high porosity, stiffness and compliance they are useful for construction e.g. of filters or crash absorbers. Foams are characterized by polyhedral cells forming a space-filling structure. They are classified as either “open-cell” with a strut-like morphology forming a continuous network, or as “closed-cell” with solid membrane-like faces. As the properties of a foam depend on the geometrical characteristics of its cells, one is interested in measuring those quantities. Classical metallographic methods using two-dimensional images of cross sections are less suitable for foams. Due to their high porosity, the preparation of sections is extremely complicated and can destroy the features of the microstructure. On the other hand, using computer microtomography, high quality three-dimensional images of foam structures are available. Various tools for deriving the desired values from these images have been developed during the last years.

GEOMETRIC MODELS FOR FOAMS

Open foams can be interpreted as the edge system of a spatial tessellation. To reproduce properties of real foams, this tessellation should be macroscopically homogeneous but microscopically heterogeneous and feature mainly pentagonal faces. Three models are studied here, two stochastic and one deterministic.
The two stochastic ones both belong to the class of Voronoi tessellations. Randomly placed seeds start to grow at the same time and at uniform rate. The growth stops when two cells meet (Okabe et al., 2000). The easiest and best known model is the Voronoi tessellation with respect to a homogeneous Poisson point process (see e.g. Stoyan et al., 1995). It is achieved using independently uniformly distributed seeds. However, the cells of a Poisson Voronoi tessellation do not show the degree of regularity and the tendency to pentagonal faces observed in real data. More regular cells are obtained if the seeds satisfy some hard core condition, i.e. there are no points closer than a fixed distance from each other. Here, the Voronoi tessellation with respect to a Matern hard core point process (Ohser and Mücklich, 2000) is used. However, it does not overcome this problem to a satisfying degree. Some deterministic models seem to be more suitable for capturing the regularity of open foams. Here, the Weaire-Phelan foam (Weaire, 1996) is used whose cells are pentagonal dodecahedra and tetradeahedra having two hexagonal and 12 pentagonal faces. However, the regularity of these models is paid by the complete loss of microscopic heterogeneity.

![Reconstructed tomographic images of a polymer foam (resolution 5 μm) and an open nickel foam (resolution 10 μm), visualization of the edge system of a Poisson Voronoi tessellation](image1)

Figure 1: Reconstructed tomographic images of a polymer foam (resolution 5 μm) and an open nickel foam (resolution 10 μm), visualization of the edge system of a Poisson Voronoi tessellation

ANALYSIS USING MINKOWSKI FUNCTIONALS

For analysis of foam structures, a basic set of characteristics from integral geometry is used - the Minkowski functionals (or quermass integrals or inner volumes). In three-dimensional space, these are volume, surface area, integral of mean curvature, and integral of total curvature, also called Gaussian curvature. For homogeneous structures, these functionals are completely determined by their densities, which are
- $V_V$ - the volume density (the volume faction or the specific volume)
- $S_V$ - the surface density (the specific surface area)
- $M_V$ - the density of the integral of mean curvature (the specific integral of mean curvature)
- $K_V$ - the density of the integral of total curvature (the specific integral of total curvature).
Details can be found in Ohser and Mücklich (2000), Schneider and Weil (2000) or Stoyan et al. (1995).

The analysis starts with estimating the densities of the Minkowski functionals from a three-dimensional image of the edge system of an open foam. From these values, other characteristics can be deduced. For open foams, the length density of the edge system $L_V = M_V / (\pi (1 - V_V))$ is of particular interest. To obtain further interesting characteristics, such as the intensity or the mean size of the cells, model assumptions have to be introduced. Knowing the geometry of the typical cell of the foam model under consideration, these mean values can
then be calculated from the measured value of $L_V$. For the Voronoi tessellations this is shown in Ohser and Mücklich (2000), for the Weaire-Phelan foam see Sych (2004).

In the present work, the dependency of the obtained mean values on the choice of the model assumptions is studied. For that purpose, the edge system of a Poisson Voronoi tessellation is analyzed using Poisson Voronoi (PV), hard core Voronoi (HCV) and Weaire-Phelan (WP) model assumptions. Because of their irregularity, Poisson Voronoi cells show a different geometry than cells appearing in real foams. Nevertheless, they have been chosen, since the characteristics of their typical cells can be obtained analytically, thus making it possible to evaluate the measured values.

30 realizations of a Poisson Voronoi tessellation with an intensity of 100 cells per volume unit have been analyzed. Table 1 shows the mean results compared to the theoretical values. The same is done for a Voronoi tessellation with respect to a Matern hard core point process of intensity 100. The hard core parameter has been chosen as $h = 0.051$ according to Ohser and Mücklich, 2000. Results are shown in Table 2. While the quality of the results depends on the resolution of the images, it is independent of the chosen intensity. As a side effect of this small study, it turned out that the error due to model choice is smaller than the error due to the choice of a “wrong” resolution. Criteria for an optimal resolution still have to be found. Here, an image of 406 resp. 350 pixels side length has been used for the unit cube in the Poisson resp. hard core case, which leads to satisfying results.

Table 1: Results for Poisson Voronoi tessellation (PV) using PV, HCV and WP model assumptions

<table>
<thead>
<tr>
<th></th>
<th>Theoretical value</th>
<th>PV</th>
<th>Deviation [%]</th>
<th>HCV</th>
<th>Deviation [%]</th>
<th>WP</th>
<th>Deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean number of cells per volume unit</td>
<td>100</td>
<td>99.88</td>
<td>-0.12</td>
<td>102.66</td>
<td>+2.66</td>
<td>114.54</td>
<td>+14.54</td>
</tr>
<tr>
<td>Mean cell volume</td>
<td>0.01</td>
<td>1.0013e-2</td>
<td>+0.13</td>
<td>9.741e-3</td>
<td>-2.59</td>
<td>8.732e-3</td>
<td>-12.68</td>
</tr>
<tr>
<td>Mean cell diameter</td>
<td>0.3141</td>
<td>0.3142</td>
<td>+0.03</td>
<td>0.3076</td>
<td>-2.07</td>
<td>0.3060</td>
<td>-2.58</td>
</tr>
<tr>
<td>Mean area of faces</td>
<td>0.01741</td>
<td>0.01741</td>
<td>0.00</td>
<td>0.01696</td>
<td>-2.58</td>
<td>0.01695</td>
<td>-2.64</td>
</tr>
</tbody>
</table>

Table 2: Results for hard core Voronoi tessellation (HCV) using PV, HCV and WP model assumptions

<table>
<thead>
<tr>
<th></th>
<th>Theoretical value</th>
<th>PV</th>
<th>Deviation [%]</th>
<th>HCV</th>
<th>Deviation [%]</th>
<th>WP</th>
<th>Deviation [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean number of cells per volume unit</td>
<td>100</td>
<td>96.71</td>
<td>-3.29</td>
<td>99.41</td>
<td>-0.59</td>
<td>110.9</td>
<td>+10.9</td>
</tr>
</tbody>
</table>
ANALYSIS USING RECONSTRUCTION OF CELLS

In order to derive empirical distributions of size, shape or other features of the cells, the cells have to be reconstructed using image analysis tools. First, the edge system of the foam is segmented. To the resulting black-and-white image, a Euclidean distance transform is applied assigning each background pixel its distance to the edge system. Ideally, this yields local maxima exactly at the cell centers. In practice, superfluous local maxima have to be removed using filters or morphological transformations. Finally, the watershed algorithm divides the inverted distance image into cells. This method has been applied to a 3d image of the edge system of a Poisson Voronoi tessellation. Sections of the resulting 3d images are shown in Figure 2.

Figure 2: 2d sections of the edge system of a Poisson Voronoi tessellation (upper left), its cells (upper right), the distance image (lower left) and the reconstructed cells (lower right)

In the following, both methods are applied to the edge system of a Poisson Voronoi tessellation with an intensity of 500 cells per volume unit. Resolution is chosen as 1/695 for analysis using Minkowski functionals and 1/412 for reconstruction. A median filter is used to eliminate superfluous maxima in the distance image. For avoidance of edge effects, only cells not intersecting the boundary are used for statistics. This causes size dependent sampling. Therefore, a minus sampling correction (Miles-Lantuejoul correction, Serra, 1982) is applied. Nevertheless, the values for the volumes returned by this method fall below the ones obtained using Minkowski functionals. This might be due to over segmentation of the watershed image arising from the irregularity of cells. While the original image contains 262 non-boundary cells, reconstruction gives 322 cells not intersecting the boundary. For real foams, where cells are much more regular, reconstruction works better.
Table 3: Results from 3d cell reconstruction compared to analysis using Minkowski functionals

<table>
<thead>
<tr>
<th></th>
<th>Theoretical value</th>
<th>Minkowski functionals</th>
<th>Cell reconstruction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean cell volume</td>
<td>0.0020</td>
<td>0.0020</td>
<td>0.0016</td>
</tr>
<tr>
<td>Mean cell diameter</td>
<td>0.1837</td>
<td>0.1841</td>
<td>0.1870</td>
</tr>
</tbody>
</table>

Results for the distribution of cell volumes are shown in Figure 3. In order to correct errors due to size dependent sampling, the Miles-Lantuejoul correction is used again.

Figure 3: Cell volume distribution of reconstructed cells

DISCUSSION

Volume image analysis is a promising tool for gaining insight into the microstructure of foams. In this paper, a method for measuring cell characteristics of an open foam from a three-dimensional image has been described and evaluated using a Poisson Voronoi tessellation. However, several fields for further research arise. A better understanding of resolution effects might lead to a recommendation concerning the choice of resolution. Here, smoothing has been used in order to remove superfluous maxima from the distance image. This holds the danger of losing small cells. A better choice for reduction of over segmentation would be using the watershed algorithm with markers obtained by complex morphological transformations. However, this is difficult for non-spherical cells. Finally, the field of model choice still holds several open questions. For example, it would be interesting to use the methods introduced here on a more regular structure, which is closer to the cells of a real foam. Possible candidates might be a random version of the Weaire-Phelan foam or random
Laguerre tessellations. But for the time being, formulas for the cell characteristics of those structures are not available.

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REFERENCES