3D characterization, modeling, and effective thermal conductivity of open aluminum foams

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1 Introduction

Foams made of aluminum or aluminum alloys are used in many application areas, e.g. as heat exchangers, catalysts or in lightweight construction. Physical properties of a foam such as thermal conductivity or elasticity are heavily influenced by its microstructure. Therefore, an understanding of the change of these properties through the microstructure is crucial for the choice of optimal foams for given applications. The use of foam models is a powerful tool for studying these relations.

Systems of edges or facets of random tessellations are often used as models for open or closed foams, respectively. The models are fitted to the microstructure of the foam cells using geometric characteristics which are estimated from tomographic images. Changing the model parameters, foams with a slightly modified microstructure can be generated. Numerical simulations in these model foams allow for an investigation of relations between the geometric structure of a material and its physical properties. Using this approach an optimization of foams for particular applications and a “virtual” design of new materials is possible.

In this paper, the cell structure of an open aluminum foam is analyzed using a tomographic image of the material. Based on the estimated characteristics a Laguerre tessellation model, i.e. a weighted Voronoi tessellation, is fitted to the foam structure. Starting from the edge system of this tessellation, the varying local thickness of the struts is reproduced using locally adaptable morphology. Finally, heat transfer is simulated in both the original image and images of various model structures and the results are compared.

2 Statistical analysis of the foam

The material under consideration is an open aluminum foam used for instance in heat exchangers or composite materials. The image has a size of 820 x 820 x 278 voxels with a voxel size of 64.57 µm. Hence, the sample corresponds to 52.95 mm x 52.95 mm x 17.95 mm of material. A visualization of the tomographic image is shown in Figure 1.

![Figure 1: Visualization (left) and section of the reconstructed cells (right) of the open aluminum foam.](image-url)
In order to estimate geometric characteristics of the foam cells, they are reconstructed using the image processing chain described in [7]. The image processing is carried out on volume images using the MAVI software package developed at the Fraunhofer ITWM [4]. From the resulting label image, a slice of which is shown in Figure 1, geometric characteristics of the single foam cells can be estimated using MAVI's ObjectFeatures functionality. Since the foam cells are slightly elongated along the z-axis, the voxel spacing in this direction is scaled by a factor of 0.92 which yields an isotropic structure. After a minus-sampling edge correction, 1038 cells with a total volume of 20940.16 mm\(^3\) are included in the statistics. The estimated mean values and standard deviations of the volume \(v\), the surface area \(s\), the diameter (mean width) \(d\), and the number of facets \(f\) of the foam cells are shown in Table 1.

### 3 Modeling

A model for the aluminum foam is obtained using the procedure introduced in [7]: In a first step, the skeleton of the foam is modelled by the edge system of a random Laguerre tessellation. Then, this edge system is dilated to the correct volume fraction of material. In order to reproduce the locally varying strut thickness, we use the concept of locally adaptable morphology: the size of the structuring element is chosen depending on the distance to the vertices.

#### 3.1 Modeling of the skeleton

Edge systems of random tessellations are widely used as models for open cell foams. The most common models are Voronoi tessellations generated by Poisson or hard core point processes [9]. However, the range of cell patterns which can be realized by these model classes is limited. Therefore, weighted generalizations of the Voronoi model are frequently considered. In particular, random Laguerre (or power) tessellations are promising models for the geometry of foam structures [6]. They are defined as follows:

Denote by \(s(x,r)\) a sphere in \(\mathbb{R}^d\) with center \(x \in \mathbb{R}^d\) and radius \(r \geq 0\) and let \(S\) be a locally finite set of spheres, which means that each bounded subset of \(\mathbb{R}^d\) is intersected only by a finite number of spheres. The Laguerre cell \(C(s(x,r),S)\) generated by a sphere \(s(x,r)\) consists of all points \(y \in \mathbb{R}^d\) which are closest to \(s(x,r)\) with respect to the so-called power distance \(\text{pow}(y, s(x,r)) = d(y,x)^2 - r^2\), where \(d(\cdot,\cdot)\) denotes the Euclidean distance.

That means

\[
C(s(x,r), S) = \{y \in \mathbb{R}^d : \text{pow}(y, s(x,r)) \leq \text{pow}(y, s(x',r')) \text{ for all } s(x',r') \in S\}. \tag{1}
\]

The Laguerre tessellation \(L(S)\) is the set of non-empty Laguerre cells of spheres contained in \(S\). It is a space-filling system of convex polytopes. If the spheres in \(S\) do not overlap, empty cells do not occur. If all radii are equal, \(L(S)\) equals the Voronoi tessellation of the set of sphere centers.

For fitting a Laguerre tessellation model to the aluminum foam we have to decide for both a suitable point process of sphere centers and a distribution of radii. Since the foam cells are very regular in shape, a Laguerre tessellation generated by a random dense packing of spheres is an adequate choice. Further, the distribution of the cell volumes in the foam is well approximated by a normal distribution. Hence, the same distribution family is chosen for the
volumes of the generating spheres. For the simulation of the sphere packings we use the force-biased algorithm introduced in [1].

Keeping the number of cells and their total volume fixed to the values observed in the real foam, we have the volume fraction \( \varphi \) of the generating ball system and the coefficient of variation \( \text{CV} \) of the ball volumes as free parameters in the optimization. As generators for tessellation candidates we simulate ball packings containing 1038 balls with volume fractions of \( \varphi = 30, 40, 50, \) and \( 60\% \) in a cube of volume 20940.16 mm\(^3\). The \( \text{CV} \) values are chosen between 0.10 and 0.25 at steps of 0.05. The corresponding Laguerre tessellations are computed using the algorithm described in [10]. In order to avoid finite size effects we use periodic boundary conditions. Four realizations are generated for each set of parameters.

The deviation of the models from the foam sample is measured using the relative distance measure

\[
D(m, c) = \sqrt{\sum_{i=1}^{8} \left( \frac{m_i - c_i}{c_i} \right)^2},
\]

where the eight entries of \( c = (c_1, \ldots, c_8) \) and \( m = (m_1, \ldots, m_8) \) are the means and standard deviations of the volume \( v \), surface area \( s \), number of facets \( f \), and diameter (mean width) \( d \) of the cells of the original foam and the model, respectively. The parameters of the best fit model with a distance value \( D_{\min} = 0.198 \) are \( \varphi = 60\% \) and \( \text{CV} = 0.15 \). The characteristics of the corresponding tessellation and their deviations from the original values are shown in Table 1. The second best model with the parameters \( \varphi = 40\% \) and \( \text{CV} = 0.20 \) yields a distance value of \( D = 0.208 \) and is included in the following steps for comparison.

<table>
<thead>
<tr>
<th></th>
<th>scaled data</th>
<th>model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean ([mm^3])</td>
<td>sd ([mm^3])</td>
</tr>
<tr>
<td>( v )</td>
<td>20.174</td>
<td>2.751</td>
</tr>
<tr>
<td>( s )</td>
<td>41.015</td>
<td>3.671</td>
</tr>
<tr>
<td>( d )</td>
<td>3.649</td>
<td>0.174</td>
</tr>
<tr>
<td>( f )</td>
<td>13.838</td>
<td>1.203</td>
</tr>
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</table>

### 3.2 Modeling of the edge thickness

In the second modeling step the edge system of the tessellation is dilated to the volume fraction observed in the original material. The locally varying thickness of the struts in the aluminum foam is reproduced using the concept of locally adaptable morphology which is a generalization of classical morphology to structuring elements of varying size. The size of the structuring element – a ball in our case – is read from a so-called size map which is an additional input for the morphological transform. It is a second image of the same size whose voxel values give the radius of the balls to be used at this location. For theoretical foundations see [3], an efficient algorithm is described in [2].

In our case the size map should mirror the fact that the struts of the real foam are thicker at the vertices than at their centers. Therefore, the edge system of the tessellation model is dilated with a ball whose radius depends on the distance to the nearest vertex. An illustration is shown in Figure 2. The computation of the size map uses several parameters governing the
local thickness at both the vertices and the edge centers. For details on this point and on the parameter choice we refer to [7].

Figure 2: The original foam structure, the edge system of the model, a classical and a locally adaptable dilation of the edge system (from left to right).

Based on the strut thickness observed for the real foam, suitable parameter ranges are determined. Altogether, 72 different parameter sets are considered for the dilation. The fit of the model is measured by the deviation of the volume fraction $V_V$ and the specific surface area $S_V$ from the values observed for the real foam. The dependence of the heat transport on $V_V$ is obvious, $S_V$ is used for further characterization of the structure. For the estimation of the characteristics, we use the FieldFeatures function of MAVI. Three of the simulated structures yield deviations in $V_V$ and $S_V$ below 1%. The exact values are given in Table 2, visualizations of the models are shown in Figure 3.

Figure 3: Visualizations of the original foam (left) and realizations of the model structures: model 1, model 2, and model 3 (from left to right).

Table 2. Estimated volume density $V_V$ and specific surface area $S_V$ of the aluminum foam and the fitted model structures.

<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>model 1</th>
<th>deviation</th>
<th>model 2</th>
<th>deviation</th>
<th>model 3</th>
<th>deviation</th>
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<tr>
<td>$V_V$ [%]</td>
<td>14.624</td>
<td>14.546</td>
<td>-0.5</td>
<td>14.517</td>
<td>-0.7</td>
<td>14.504</td>
<td>-0.8</td>
</tr>
<tr>
<td>$S_V$ [1/mm]</td>
<td>840.223</td>
<td>847.861</td>
<td>+0.9</td>
<td>845.860</td>
<td>+0.7</td>
<td>844.289</td>
<td>+0.5</td>
</tr>
</tbody>
</table>

4 Simulation of effective heat transport coefficients in porous materials

The modeling of the heat flow through porous materials follows closely the experimental set up as shown in Figure 4.
The porous structure is fixed between a hot plate at temperature $T_{in}$ and a cold plate at temperature $T_{out}$. A perfect heat transfer is assumed between the plates and the porous structure and the measurement monitors various energy losses and the electrical heating and cooling of the plates. From the energy balances, the heat flux through the porous material may be calculated and from that, the effective heat transport coefficient is found.

In our simulation, the heat flow is modeled by the heat equation

$$\frac{\partial}{\partial t} T = \kappa \Delta T$$

with the thermal diffusivity $\kappa$, temperature $T$, and $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. We use isolating Neumann boundary conditions on the porous structure and constant temperature Dirichlet conditions on the top and bottom metal plates. This equation is discretized by standard finite volumes and solved in the steady state.

Using standard harmonic analysis, the effective heat conductivity $\lambda_{eff}$ is the given by

$$\lambda_{eff} = \frac{W_{in} - W_{out}}{d / 2 \cdot A}.$$  

$W_{in}$ and $W_{out}$ denote the heat flow induced by the constant temperature Dirichlet boundary conditions.

The implementation of the heat flow solver is fully parallelized on pc clusters using the MPI [8]; on cluster nodes, we take full advantage of multi-core architectures by using a thread based parallelization. Even the portability to up coming specialized number crunching architectures, like the Cell blades of IBM, has been demonstrated [5].

5 Simulation results and discussion

In a first step, $\lambda_{eff}$ is simulated in both the binarized tomographic image of the real foam and the three model structures chosen in Section 3.2. Three realizations are considered for each model which yields mean values of $\lambda_{eff,1} = 15.48$ W/m/K, $\lambda_{eff,2} = 15.45$ W/m/K, and $\lambda_{eff,3} = 15.48$ W/m/K, which are in good agreement with the value for the real structure ($\lambda_{eff,real} = 15.65$ W/m/K). For comparison, the effective heat conductivity is also simulated for some of the model structures obtained during the optimization procedure described in Section 3.2. Furthermore, we generated dilated versions of the edge system of a Poisson Voronoi tessellation with the same number of cells as in the real foam. For both models, also the
classical dilation, i.e. with a ball of fixed size, is considered for comparison. The results are shown in Figure 5.

![Figure 5: Left: Simulated effective heat conductivity $\lambda_{\text{eff}}$ plotted versus $V_V$ for model structures based on the edge systems of Laguerre tessellations generated by force-biased packings (FB) and Poisson Voronoi tessellations (PV). Examples of both classical and locally adaptable dilations are shown. Right: Visualizations of the two model structures producing the black circles in the plot: $V_V = 13.58\%$, $S_V = 792.58 \text{ 1/mm}$, $\lambda_{\text{eff}} = 12.75 \text{ W/m/K}$ (left), and $V_V = 13.48\%$, $S_V = 849.47 \text{ 1/mm}$, $\lambda_{\text{eff}} = 14.05 \text{ W/m/K}$ (right).]

The dependence of $\lambda_{\text{eff}}$ on the volume fraction is clearly visible. However, the example of the two structures visualized in Figure 5 indicates that further characteristics besides $V_V$, in particular the degree of curvature of the edges, play a role for the heat transport. This geometric feature can be studied e.g. by means of the geometric tortuosity or the spherical granulometry distribution [7]. For surface sensitive processes the variation between the models should be more pronounced. A detailed investigation of this point is subject to further research.

7 References