Inflation-Linked Products and Optimal Investment With Macro Derivatives

In this thesis diverse problems concerning inflation-linked products are dealt with. To start with, two models for inflation are presented, including a geometric Brownian motion for consumer price index itself and an extended Vasicek model for inflation rate. For both suggested models the pricing formulas of inflation-linked products are derived using the risk-neutral valuation techniques. As a result Black and Scholes type closed form solutions for a call option on inflation index for a Brownian motion model and inflation evolution for an extended Vasicek model as well as for an inflation-linked bond are calculated. These results have been already presented in Korn and Kruse (2004) [17]. In addition to these inflation-linked products, for the both inflation models the pricing formulas of a European put option on inflation, an inflation cap and floor, an inflation swap and an inflation swaption are derived. Consequently, basing on the derived pricing formulas and assuming the geometric Brownian motion process for an inflation index, different continuous-time portfolio problems as well as hedging problems are studied using the martingale techniques as well as stochastic optimal control methods. These utility optimization problems are continuous-time portfolio problems in different financial market setups and in addition with a positive lower bound constraint on the final wealth of the investor. When one summarizes all the optimization problems studied in this work, one will have the complete picture of the inflation-linked market and both counterparts of market-participants, sellers as well as buyers of inflation-linked financial products. One of the interesting results worth mentioning here is naturally the fact that a regular risk-averse investor would like to sell and not buy inflation-linked products due to the high price of inflation-linked bonds for example and an underperformance of inflation-linked bonds compared to the conventional risk-free bonds. The relevance of this observation is proved by investigating a simple optimization problem for the extended Vasicek process, where as a result we still have an underperforming inflation-linked bond compared to the conventional bond. This situation does not change, when one switches to an optimization of expected utility from the purchasing power, because in its nature it is only a change of measure, where we have a different deflator. The negativity of the optimal portfolio process for a normal investor is in itself an interesting aspect, but it does not affect the optimality of handling inflation-linked products compared to the situation not including these products into investment portfolio. In the following, hedging problems are considered as a modeling of the other half of inflation market that is inflation-linked products buyers. Natural buyers of these inflation-linked products are obviously institutions that have payment obligations in the future that are inflation connected. That is why we consider problems of hedging inflation-indexed payment obligations with different financial assets. The role of inflation-linked products in the hedging portfolio is shown to be very important by analyzing two alternative optimal hedging strategies, where in the first one an investor is allowed to trade as inflation-linked bond and in the second one he is not allowed to include an inflation-linked bond into his hedging portfolio. Technically this is done by restricting our original financial market, which is made of a conventional bond, inflation index and a stock correlated with inflation index, to the one, where an inflation index is excluded. As a whole, this thesis presents a wide view on inflation-linked products: inflation modeling, pricing aspects of inflation-linked products, various continuous-time portfolio problems with inflation-linked products as well as hedging of inflation-related payment obligations.