Limit theorems constitute a classical and important field in probability theory. In several applications, in particular in demographic or medical contexts, killed Markov processes suggest themselves as models for populations undergoing culling by mortality or other processes. In these situations mathematical research features a general interest in the observable distribution of survivors, which is known as Yaglom limit or quasi-stationary distribution. Previous work often focuses on discrete state spaces, commonly birth-death processes (or with some more flexible localization of the transitions), with killing only on the boundary. The central concerns of this thesis are to describe, for a given class of one dimensional diffusion processes, the quasi-stationary distributions (if any), and to describe the convergence (or not) of the process conditioned on survival to one of these quasi-stationary distributions. Rather general diffusion processes on the half-line are considered, where 0 is allowed to be regular or an exit boundary. Very similar techniques are applied in this work in order to derive results on the large time behavior of an exotic measure valued process, which is closely related to so-called point interactions, which have been widely studied in the mathematical physics literature.