Parallel schedulers on dense matrices

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Preconditions I

- Using **dense matrices** with **unsigned int64** entries.
- Computing in F_p, p some **prime** $< 2^{16}$.
- We compared the following set of parallel schedulers:
  1. **pthread** (or in other words, by hand),
  2. **OpenMP** (sometimes together with pthread),
  3. **Intel TBB** (using lambda expressions),
  4. **XKAAPI** (in particular, the C interface KAAPIC),

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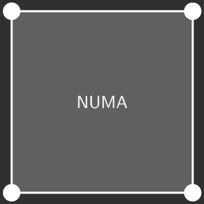
Note

The implemented algorithms are **not optimized** in order to keep the influence on the schedulers as low as possible.

Preconditions II

Results presented computed on the **HPAC compute server**



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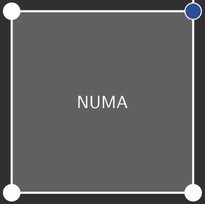

with hyperthreading: 64 **cores**

- 8 Intel Xeon E5-4620 cores @ 2.20 GHz
- L1 cache: 32 KB
- L2 cache: 256 KB
- shared L3 cache: 16 MB
- 96 GB RAM

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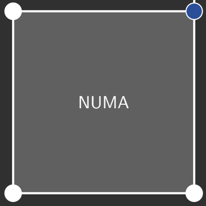

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Also tested on: 48-core (real cores) AMD Magny Cours NUMA,
4-core (8 with hyperthreading) Intel Sandy Bridge.

Tested algorithms

1. **Naive Dense Matrix Multiplication**
2. **Dense Gaussian Elimination**:
   (a) **Naive** implementation (with and without pivoting)
   (b) **Cache-oblivious** implementation (GEP by Chowdhury and Ramachandran without pivoting)

Naive dense matrix multiplication

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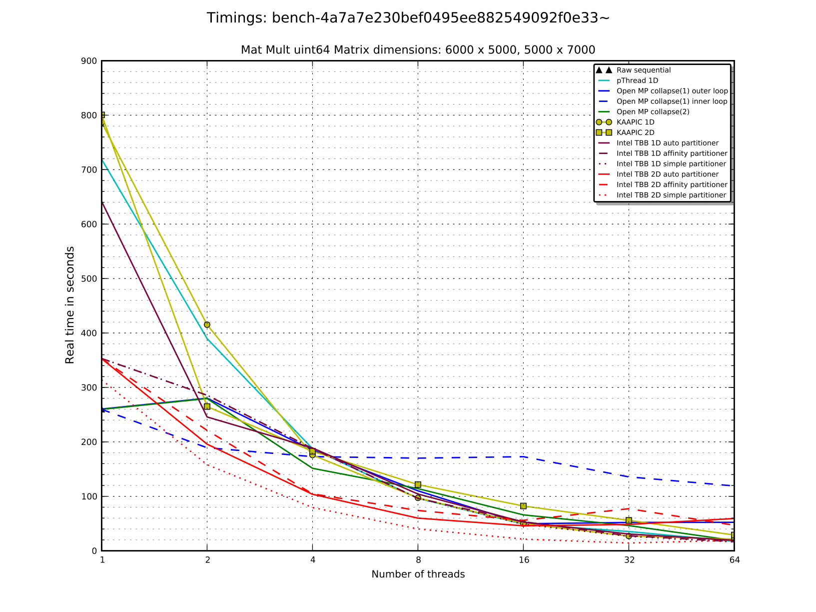
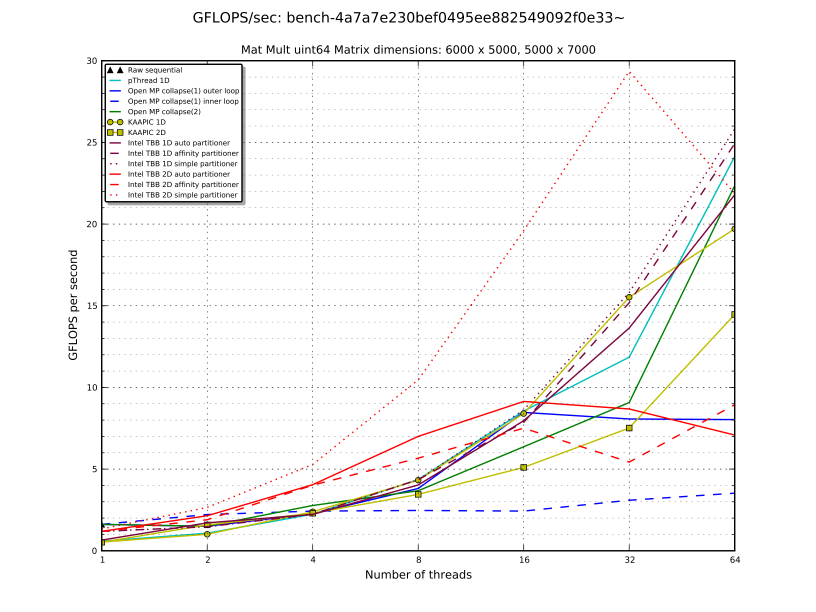
- For Intel TBB we compared the different integrated schedulers:
  - **auto partitioner:** Splitting work to balance load
  - **affine partitioner:** Improves choice of CPU affinity
  - **simple partitioner:** Recursively splits a range until it is no longer divisible (grainsize is critical)

Timings


Timings: bench-4a7a7e230bef0495ee882549092f0e33~
Mat Mult uint64 Matrix dimensions: 6000 x 5000, 5000 x 7000
Real time in seconds
0
100
200
300
400
500
600
700
800
900
Number of threads
1
2
4
8
16
32
64
Raw sequential
pThread 1D
Open MP collapse(1) outer loop
Open MP collapse(1) inner loop
Open MP collapse(2)
KAAPIC 1D
KAAPIC 2D
Intel TBB 1D auto partitioner
Intel TBB 1D affinity partitioner
Intel TBB 1D simple partitioner
Intel TBB 2D auto partitioner
Intel TBB 2D affinity partitioner
Intel TBB 2D simple partitioner

GFLOPS/sec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GFLOPS per second
0
5
10
15
20
25
30
Number of threads
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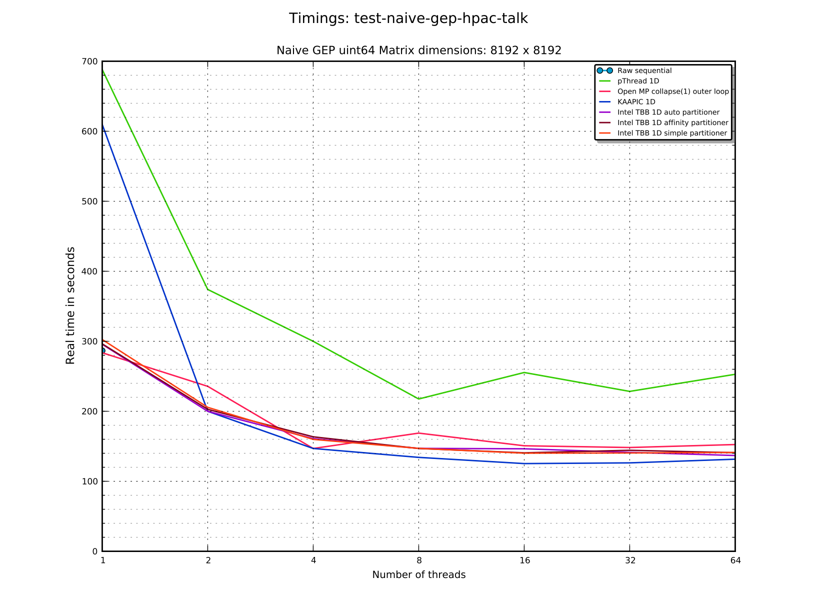
- **KAAPIC**, **Open MP** and **Intel TBB** are in the same range.
- **Open MP** behaves a bit worse when it comes to hyperthreading.
- **pthread** implementation slows down due to lack of real scheduler.

Timings



Timings: test-naive-gep-hpac-talk
Naive GEP uint64 Matrix dimensions: 8192 x 8192
Raw sequential
pThread 1D
Open MP collapse(1) outer loop
KAAPIC 1D
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Real time in seconds
Number of threads
0
100
200
300
400
500
600
700
1
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4
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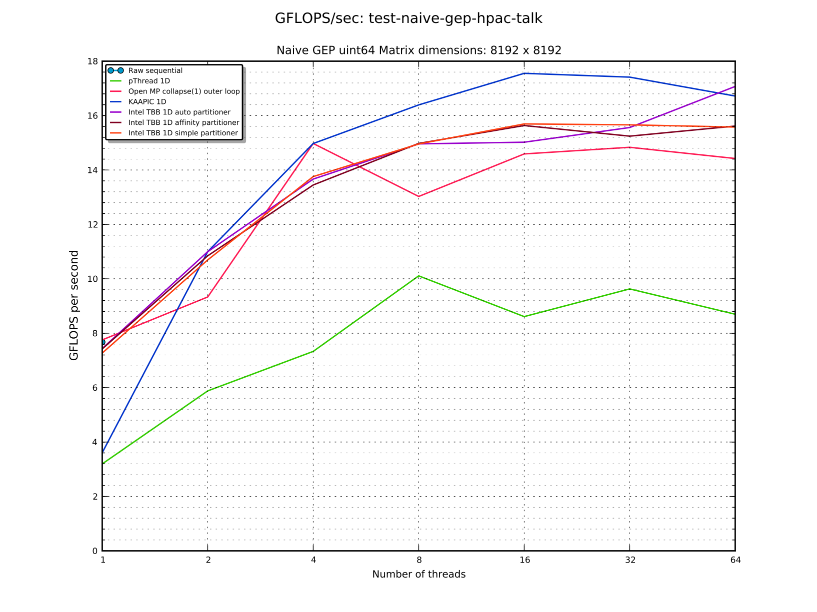
16
32
64

GFLOPS/sec



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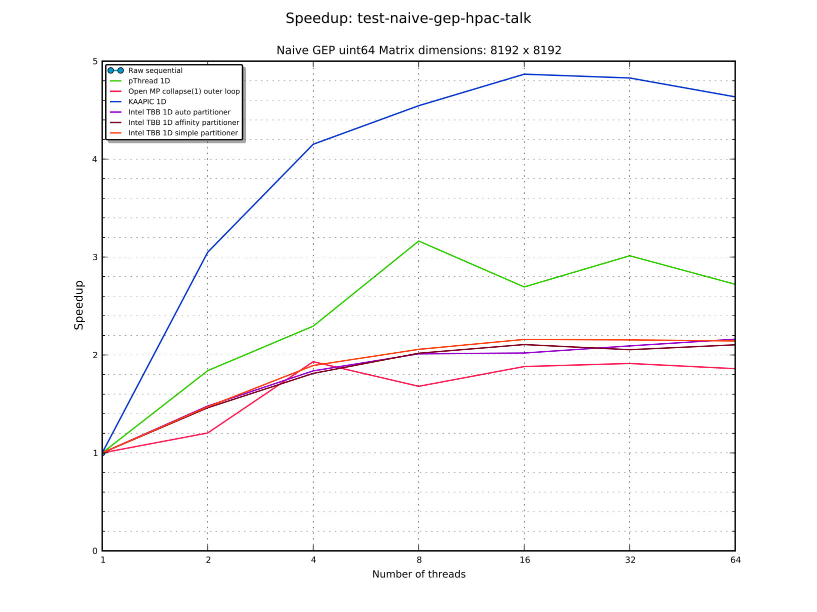
GFLOPS per second
Number of threads
0
2
4
6
8
10
12
14
16
18
1
2
4
8
16
32
64

Speedup



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Speedup
Number of threads
0
1
2
3
4
5
1
2
4
8
16
32
64

- Basics
- Naive dense matrix multiplication
- Naive dense Gaussian Eliminaton
- **Cache-oblivious dense Gaussian Elimination**
- Features of the library

Cache-oblivious dense Gaussian Elimination

Implemented **I-GEP** from [CR10].

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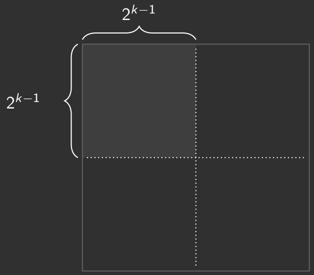

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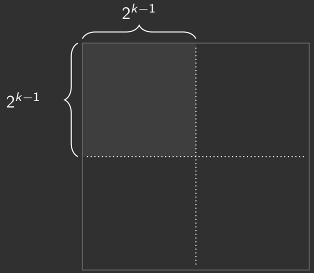
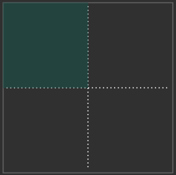


- Stop recursion once parts fit in cache.

Cache-oblivious dense Gaussian Elimination

Needs a bit of globally bookkeeping (inverse pivots, etc.)

Cache-oblivious dense Gaussian Elimination



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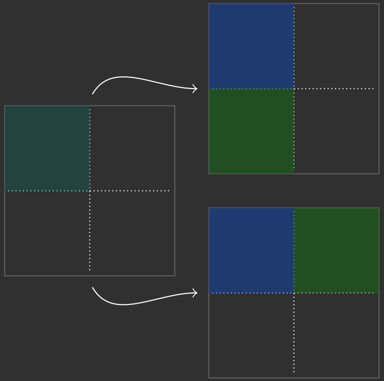

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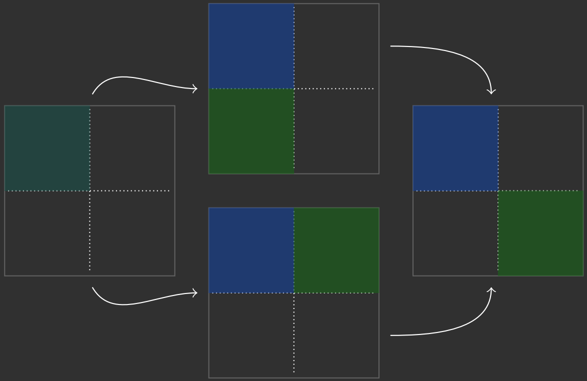

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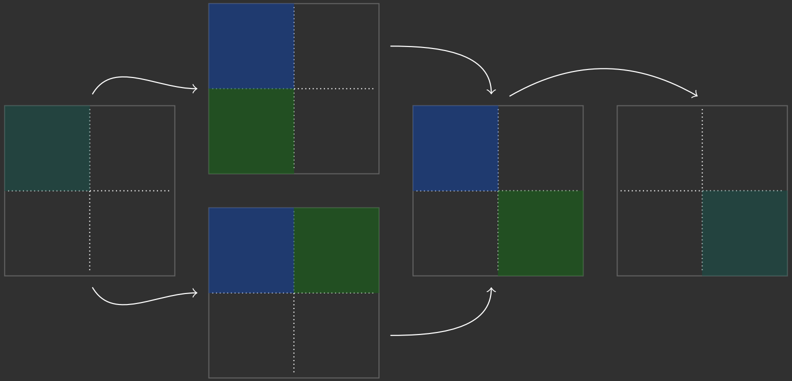

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Differences to the naive approach:

- The base cases are **not parallelized**.
- There are **no parallel FOR loops**.
- Instead we need to use a **recursive task scheduling**:
  - **pthread**: no scheduling, left unbound.
  - **Open MP**: PARALLEL SECTIONS (real tasks should be available in Open MP 4.0)
  - **KAAPIC**: KAAPIC_SPAWN
  - **Intel TBB**: INVOKE

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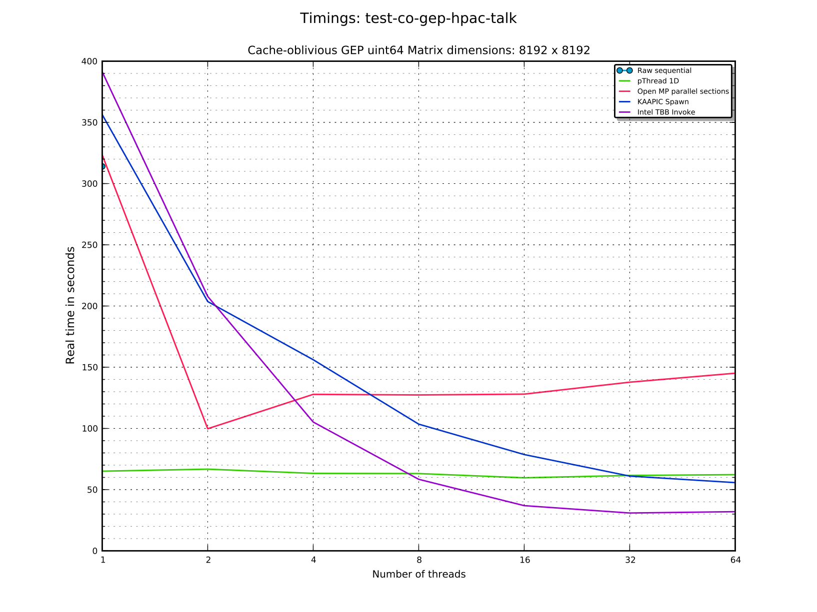

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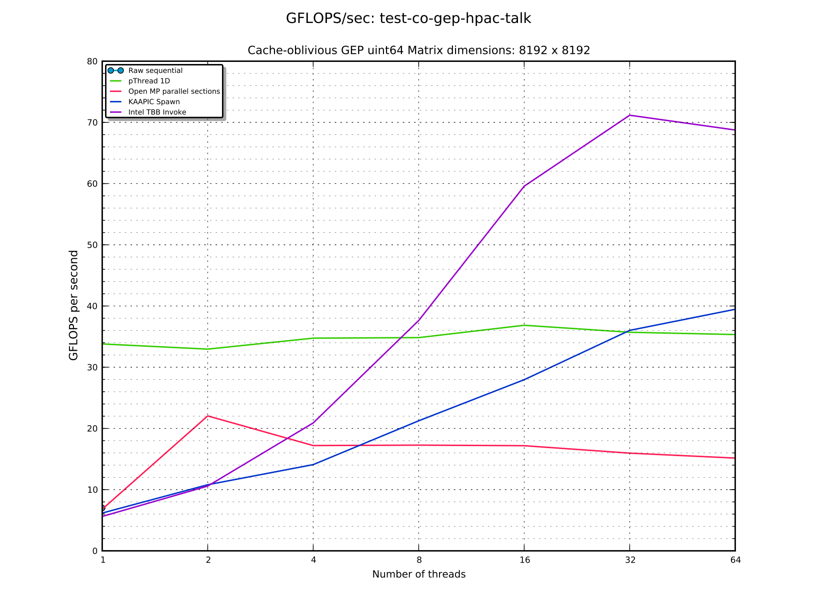

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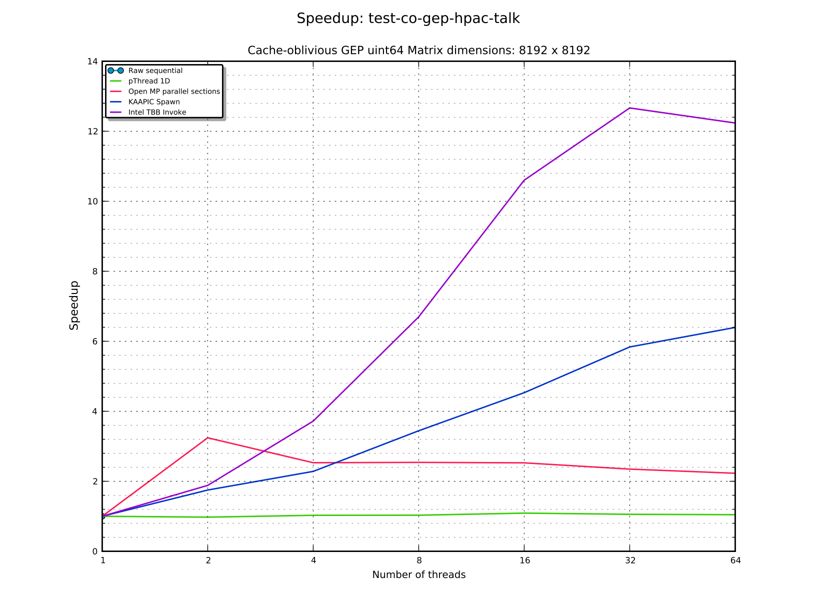

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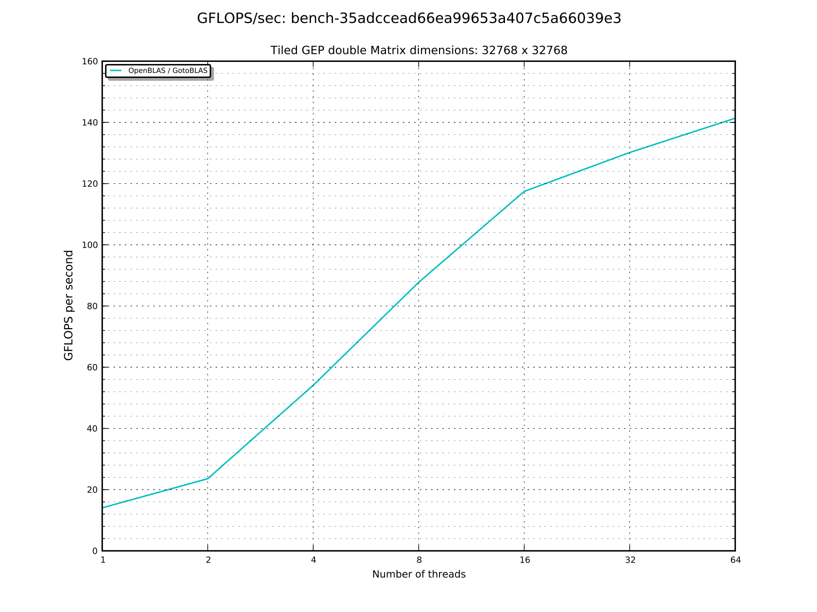

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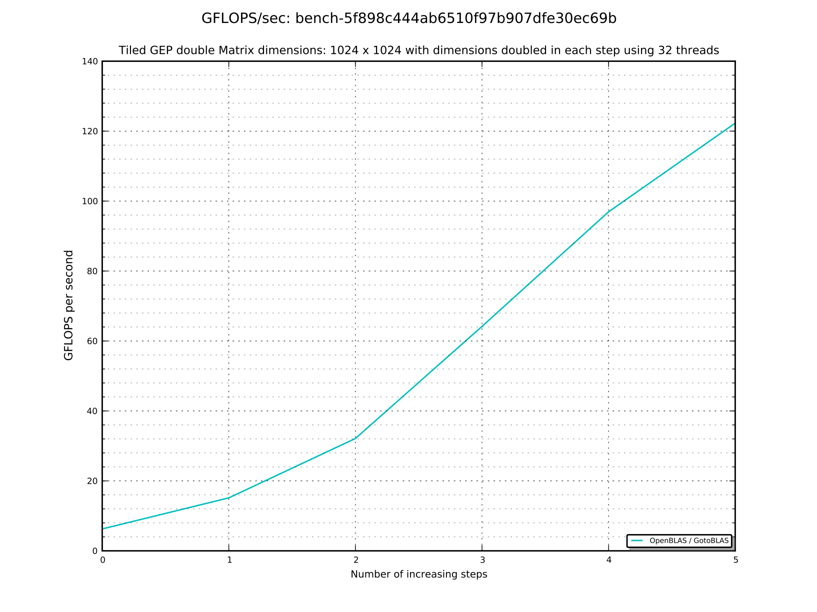

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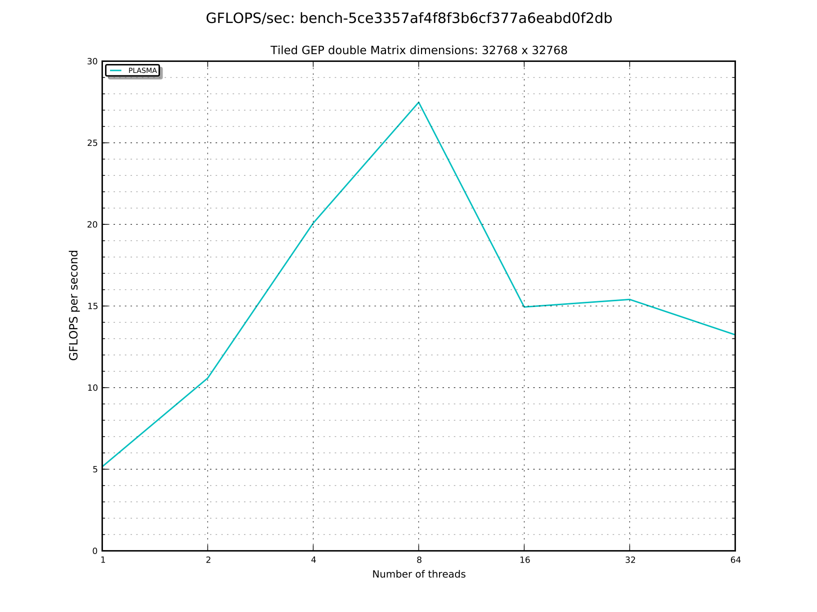

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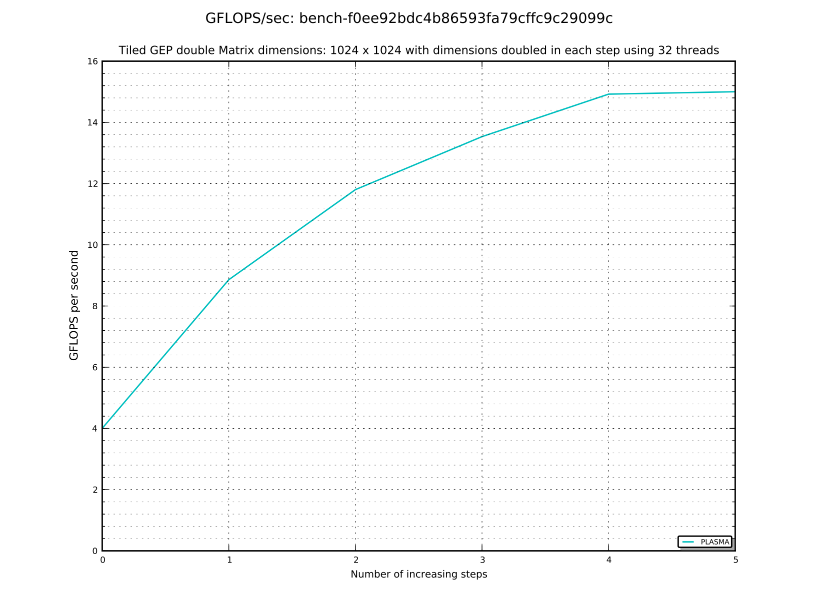

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- Publicly available: `https://github.com/ederc/LA-BENCHER`

Bibliography

[PL13] E. Agullo et al. PLASMA Users' Guide: Parallel Linear Algebra Software for Multicore Architectures, Version 2.0

[OB13] Z. Xianyi, W. Quian and Z. Chothia. OpenBLAS, `http://xianyi.github.com/OpenBLAS`

[CR10] R. A. Chowdhury and V. Ramachandran. The Cache-Oblivious Gaussian Elimination Paradigm: Theoretical Framework, Parallelization and Experimental Evaluation

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[WPD01] R. C. Whaley, A. Petitet and J. J. Dongarra Automated Empirical Optimization of Software and the ATLAS Project

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