Complements

K - algebras.

Def^N: Let K be a field.
A K-algebra is an ordered 4-tuple $(A, +, \cdot, *)$ satisfying the following axioms:

(A1) $(A, +, \cdot)$ is a ring (here: assumed to be associative)

(A2) $(A, +, *)$ is a K-vector space

(A3) $\lambda * (a \cdot b) = a \cdot (\lambda * b) = (\lambda * a) \cdot b$ $\quad \forall \lambda \in K, \forall a, b \in A.$

Def^N: A map $f: A \to B$ between two K-algebras A and B is called a(n) (K)-algebra homomorphism iff:

(a) f is K-linear

(b) f is a homomorphism of rings, i.e.
$f(a \cdot b) = f(a) \cdot f(b)$ $\forall a, b \in A$, and
$\left(f(1_A) = f(1_B) \text{ if both } A \text{ and } B \text{ are unital rings.} \right)$

Examples:

(a) the field K itself is a K-algebra.

(b) $\forall n \in \mathbb{Z}_{\geq 1}$ the $n \times n$-square matrices $M_n(K)$ form a K-algebra.

(c) $K[X_1, \dots, X_n]$ is a K-algebra

(d) $\mathbb{R}$ and $\mathbb{C}$ are $\mathbb{Q}$-algebras, $\mathbb{C}$ is an $\mathbb{R}$-algebra, ...