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# LOW-DIMENSIONAL REPRESENTATIONS OF QUASI-SIMPLE GROUPS

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*Abstract*

We determine all the absolutely irreducible representations of degree up to 250 of quasi-simple finite groups, excluding groups that are of Lie type in their defining characteristic. Additional information is also given on the Frobenius–Schur indicators and the Brauer character fields of the representations.

## 1. *Introduction*

In this paper we classify low-dimensional absolutely irreducible representations of quasi-simple groups. Results of this type seem to have many applications. We have at least one immediate application in mind, namely the matrix group-recognition program in computational group theory.

Our main result is a list of all the absolutely irreducible representations of quasi-simple groups up to degree 250 (excluding representations of groups that are of Lie type in their defining characteristic), together with some information on these representations, such as the character fields and the Frobenius–Schur indicators.

**Theorem 1.** *Let $G$ be a quasi-simple finite group, and let $V$ be an absolutely irreducible faithful $FG$-module of dimension $d \leqslant 250$, where the characteristic of $F$ is not the defining characteristic of $G$ if $G$ is of Lie type. Then the values of $(G, \dim(V))$, together with some additional information, are contained in Tables 2 and 3.*

Previous work of Kondratiev (see [**36**] and the references therein) has solved this question for representations up to degree 27. His results are recovered as part of our main theorem. In the case not considered here—that of irreducible representations of groups of Lie type in their defining characteristic—Aschbacher (in unpublished work) obtained a classification up to degree 27. Recent work by Lübeck [**37**] has determined all the low-dimensional representations of groups of Lie type in their defining characteristic, thus complementing our list. More precisely, Lübeck has determined all the absolutely irreducible representations of $G$ of degree up to at least $l^3/8$, where $l$ is the Lie rank of $G$, including all those of degree at most 250.

The construction of the list falls naturally into three parts, according to the classification of finite simple groups. If the group $G$ is of Lie type, we make use of results obtained by various authors on low-dimensional representations in cross-characteristic (see [**15, 16, 23**]), and the Landazuri–Seitz–Zalesskii bounds [**49**]. For the alternating groups and their spin-covers, results obtained by James [**27, 28**] and Wales [**55**] can be used. Finally, for the sporadic groups, there exists a wealth of published and unpublished tables of Brauer characters, computed by different authors.

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In all three classes, the tables for the smaller groups can be found in the ordinary and modular Atlas [**5, 34**], and also in the GAP computer algebra system [**8**]. Nevertheless, it turns out that in all three families there remain some groups where the theoretical results cannot be applied to determine all the candidate representations, and where the corresponding tables are also not contained in GAP. In these cases, the open questions could be solved with the help of the MOC computer system [**33**].

Throughout our paper we adopt the notation of the Atlas [**5**] for the quasi-simple groups.

## 2. *Groups of Lie type*

In this section we describe the proof of correctness of the table for groups that are of Lie type in their non-defining characteristic.

### 2.1. *Known results*

The starting point is a result of Landazuri–Seitz–Zalesskii [**49**], giving lower bounds for degrees of non-trivial representations of quasi-simple groups of Lie type in their non-defining characteristic. Using this, we get down to the following finite list of simple candidate groups whose covering groups possibly have non-trivial irreducible representations of degree at most 250:

| | | | |
|---|---|---|---|
| $\mathrm{L}_2(q)\ \ (q \leqslant 499)$, | $\mathrm{L}_3(q)\ \ (q \leqslant 13)$, | $\mathrm{L}_4(q)\ \ (q \leqslant 5)$, | $\mathrm{L}_5(2)$, |
| $\mathrm{L}_5(3)$, | $\mathrm{L}_6(2)$, | $\mathrm{L}_7(2)$, | |
| $\mathrm{U}_3(q)\ \ (q \leqslant 16)$, | $\mathrm{U}_4(q)\ \ (q \leqslant 5)$, | $\mathrm{U}_5(q)\ \ (q \leqslant 4)$, | $\mathrm{U}_6(2)$, |
| $\mathrm{U}_6(3)$, | $\mathrm{U}_7(2)$, | $\mathrm{U}_8(2)$, | $\mathrm{U}_9(2)$, |
| $\mathrm{S}_4(q)\ \ (q \leqslant 19)$, | $\mathrm{S}_6(q)\ \ (q \leqslant 7)$, | $\mathrm{S}_8(q)\ \ (q \leqslant 3)$, | $\mathrm{S}_{10}(q)\ \ (q \leqslant 3)$, |
| $\mathrm{O}_7(3)$, | $\mathrm{O}_8^+(2)$, | $\mathrm{O}_8^-(2)$, | $\mathrm{O}_8^-(3)$, |
| ${}^2B_2(8)$, | ${}^2B_2(32)$, | $G_2(q)\ \ (q \leqslant 5)$, | |
| ${}^3D_4(2)$, | ${}^3D_4(3)$, | ${}^2F_4(2)'$, | $F_4(2)$. |

From this list we can eliminate the groups whose modular character tables are contained in the *Atlas of Brauer characters* [**34**], or in GAP [**8**]. Furthermore, the irreducible representations of $\mathrm{L}_2(q)$ in non-defining characteristic are well-known (see [**3**]), and the relevant information is displayed in Table 2. For linear groups, Guralnick and Tiep [**15**] have described the representations of the first two non-trivial degrees, and have given a lower bound for the third-smallest degree. Similarly, this has been done for unitary groups by the present authors in [**23**], and for the symplectic groups over fields of odd order in [**16**]. Using these results, we are left with the following groups:

| | | | | | |
|---|---|---|---|---|---|
| $\mathrm{L}_4(4)$, | $\mathrm{L}_4(5)$, | $\mathrm{U}_4(4)$, | $\mathrm{S}_4(7)$, | $\mathrm{S}_4(8)$, | $\mathrm{S}_{10}(2)$, |
| $\mathrm{O}_8^-(3)$, | ${}^3D_4(3)$ | $F_4(2)$. | | | |

Restriction to $2.\mathrm{O}_7(3)$ (which is in GAP) shows that $2.\mathrm{O}_8^-(3)$ has no faithful characters of degree at most 250, and the characters of the simple group $\mathrm{O}_8^-(3)$ are in GAP.

### 2.2. *Some groups of Lie type and MOC*

Here we comment on the computation of the results for some of the smaller groups of Lie type that are not contained in the Modular Atlas. For the linear groups $\mathrm{L}_4(4)$ and $\mathrm{L}_4(5)$, we used the modular representation theory of the general linear groups in the non-defining characteristic case, as developed by Dipper and James in a series of papers. In particular, we consulted reference [**29**], in which James computed the decomposition matrices for $\mathrm{GL}_n(q)$ up to $n = 10$.

For the unitary group $U_4(4)$, we used the information contained in the diploma thesis of Wings [61], as well as the generic character table of $U_4(q)$ computed by Nozawa [44]. Partial decomposition matrices for the 4-dimensional symplectic groups have been obtained by White, and completed by Okuyama and Waki in [45]. For the case of cyclic defect groups, the decomposition matrices have been computed by Fong and Srinivasan [7]. To deal with the groups $S_4(7)$ and $S_4(8)$, we used White's papers [56, 57, 58]. For the character values, we consulted the generic character tables (see [50], [46] and [6]). The desired result for ${}^3D_4(3)$ can be extracted from Geck's work [9], and that for $F_4(2)$ is contained in [20].

The decomposition matrices for $F_4(2)$, as well as those for $S_{10}(2)$ and many sporadic groups, have been computed with the help of MOC [33], a system for dealing with modular characters. This was developed by Jansen, Lux, Parker and the first author, beginning in 1984. A description of MOC and its algorithms is given in [33], which unfortunately never found a publisher. Some of the algorithms and methods used are also described in [38, Section 3] and in [39, Section 3].

Let us briefly sketch the principal idea, without going into too many details. Let $G$ be a finite group, and let $p$ be a prime dividing the order of $G$. Suppose that the ordinary character table of $G$ and the $p$-modular character tables of some (maximal) subgroups of $G$ are known. In a first step, we compute a large set $B$ of $p$-modular Brauer characters, and a large set $P$ of projective characters of $G$. This is done using the usual methods, by tensoring characters, or by inducing them from subgroups.

Let $\mathrm{IBr}(G)$ and $\mathrm{IPr}(G)$ denote the set of irreducible Brauer characters and the set of projective indecomposable characters of $G$, respectively. In a second step, we compute a basic set of Brauer characters BS from $B$, and a basic set of projective characters PS from $P$. The term 'basic set' here means that the transition matrices $X$ and $Y$, with $\mathrm{BS} = X \cdot \mathrm{IBr}(G)$ and $\mathrm{PS} = Y \cdot \mathrm{IPr}(G)$, are unimodular and have non-negative integral entries. (In the above matrix equations, we have identified the sets of characters with their matrices of values, so that, for example, '$\mathrm{IBr}(G)$' also stands for the Brauer character table of $G$.)

Let $U$ denote the square matrix of scalar products between the characters in BS and those in PS. Then $U = XY^t$, by the orthogonality relations. Thus, in order to find candidates for $\mathrm{IBr}(G)$, one has to find the factorizations of $U$ into a product of unimodular matrices with non-negative integral entries. If $U = X_1 Y_1^t$ is such a factorization, then $X_1^{-1} \cdot \mathrm{BS}$ is a candidate set for the irreducible Brauer characters. The projective characters in $P$ reduce the number of candidates for $\mathrm{IBr}(G)$, and thus for the first factor $X_1$, since every irreducible Brauer character must have a non-negative scalar product with every character in $P$. Dually, the Brauer characters in $B$ restrict the number of possibilities for the second factor, $Y_1^t$.

Of course, in each individual problem we used some ad-hoc techniques, adapted to the particular problem. The application of these techniques was supported by substantial calculations, made using GAP and MAPLE [4].

## 3. *The alternating groups*

In this section, we collect some results on low-dimensional representations of alternating groups and their covering groups. For this purpose, we recall some standard notation. For a field $F$ and a partition $\lambda \vdash n$, let $S^\lambda$ denote the corresponding Specht module for $\mathfrak{S}_n$ over $F$. Assume that $F$ has characteristic $\ell$. For $\ell$-regular partitions $\lambda$ we write $D^\lambda$ for the irreducible $F\mathfrak{S}_n$-module indexed by $\lambda$, the unique simple composition factor in the socle of $S^\lambda$. It is known that the $D^\lambda$ constitute a complete set of representatives for the irreducible $F\mathfrak{S}_n$-modules.

### 3.1. *2-modular representations of $\mathfrak{A}_n$*

We first consider the case of representations over fields of characteristic $\ell = 2$. The 2-modular character table of $\mathfrak{S}_n$ is known for $n \leqslant 14$ [1, 34].

**Proposition 2.** *Let $n \geqslant 15$, $\ell = 2$ and $(n) \neq \lambda \vdash n$ be 2-regular. Assume that $\dim(D^\lambda) \leqslant 500$. Then one of the following cases occurs:*

(1) $\lambda = (n-1, 1)$ *and* $n \leqslant 502$, *or*

(2) $\lambda = (n-2, 2)$ *and* $n \leqslant 34$, *or*

(3) $\lambda = (12, 3)$, $\dim(D^\lambda) = 336$, *or*

(4) $\lambda = (8, 7), (9, 7)$, $\dim(D^\lambda) = 128$, *or*

(5) $\lambda = (9, 8), (10, 8)$, $\dim(D^\lambda) = 256$.

*Proof.* From the known 2-modular character table of $\mathfrak{S}_{14}$, it follows that for $n = 14$, either

$$\lambda \in M_{14} := \{(14), (13, 1), (12, 2), (11, 3), (10, 4), (8, 6)\}$$

or $\dim(D^\lambda) > 500$. Now let $n = 15$ and assume that $\dim(D^\lambda) \leqslant 500$. Then the restriction $D^\lambda|_{\mathfrak{S}_{14}}$ has to contain some $D^\mu$, with $\mu \in M_{14}$, as a submodule. This implies that $S^\lambda$ is a constituent of the induced module $(S^\mu)^{\mathfrak{S}_{15}}$, and hence that

$$\lambda \in \{(15), (14, 1), \ldots, (9, 6), (8, 7), (8, 6, 1)\}$$

by the branching rule for ordinary characters of $\mathfrak{S}_n$. The dimension of $D^\lambda$ for $\lambda$ in the above list is known by [27, Theorems 5.2 and 7.1], and it turns out that $\dim(D^\lambda) \leqslant 500$ only for

$$\lambda \in M_{15} := \{(15), (14, 1), (13, 2), (12, 3), (8, 7)\}.$$

The same argument may be applied to obtain $M_{16}, \ldots, M_{19}$, yielding in particular $M_{19} = \{(19), (18, 1), (17, 2)\}$. Now, induction and the branching rule allow us to complete the proof. The degrees of the relevant $D^\lambda$ follow from [27, Theorem 5.2]. □

Benson has determined those 2-regular partitions $\lambda = (\lambda_1 \geqslant \lambda_2 \geqslant \cdots)$ for which the irreducible $\mathfrak{S}_n$-module $D^\lambda$ splits upon restriction to the alternating group $\mathfrak{A}_n$: the restriction is irreducible if $\lambda_{2j-1} - \lambda_{2j} > 2$ or if $\lambda_{2j-1} + \lambda_{2j} \equiv 2 \pmod 4$ for some $j > 0$ [2, Theorem 1.1].

We thus obtain the following corollary to the theorem.

**Corollary 3.** *Let $n \geqslant 15$, let $\ell = 2$, and let $D$ be a non-trivial absolutely irreducible $\mathfrak{A}_n$-module of dimension $\dim(D) \leqslant 250$. Then either $D$ is the restriction to $\mathfrak{A}_n$ of either $D^{(n-1,1)}$ with $n \leqslant 252$, or $D^{(n-2,2)}$ with $n \leqslant 24$, or*

$$(n, \dim(D)) \in \{(15, 64), (16, 64), (17, 128)\}.$$

### 3.2. *$\ell$-modular representations of $\mathfrak{A}_n$, $\ell > 2$*

Now, assume that $F$ is a sufficiently large field of characteristic $\ell \neq 2$. The decomposition matrices of $\mathfrak{S}_n$ over $F$ are known for $n \leqslant 14$ (see [34]), or can be computed using the Specht-package in GAP, written by Andrew Mathas. (In fact, for $\ell \neq 3$ the decomposition matrices can be computed up to at least $n = 20$, but we will not need this here.)

We obtain the following analogue of Proposition 2.

**Proposition 4.** *Let $n \geqslant 14$, and let $M$ be a non-trivial absolutely irreducible $F\mathfrak{A}_n$-module of dimension $\dim(M) \leqslant 250$. Then $M = D^\lambda$ for $\lambda$ one of the following:*

(1) $\lambda = (n-1, 1)$ *and* $n \leqslant 252$, *or*

(2) $\lambda = (n-2, 2)$ *and* $n \leqslant 24$, *or*

(3) $\lambda = (n-2, 1^2)$ *and* $n \leqslant 24$, *or*

(4) $\lambda = (11, 3)$ *and* $\ell = 5, 7$, *or*

(5) $\lambda = (7, 7)$ *and* $\ell = 5$.

*Proof.* We can argue essentially as in the proof of the preceding proposition. First, let $\ell = 3$. From the known decomposition matrix, it follows that the irreducible $\mathfrak{A}_{13}$-modules $D$ with $\dim(D) \leqslant 250$ are the ten relevant modules $D^\lambda$ with

$$\lambda \in M_{3,13} := \{(13), (12, 1), (11, 2), (11, 1^2), (10, 3), (10, 2, 1)\}.$$

In particular, all of them are restrictions of irreducible $\mathfrak{S}_{13}$-modules. Thus, if $D$ is an irreducible $\mathfrak{A}_{14}$-module of dimension $1 \neq \dim(D) \leqslant 250$, then $D$ is a constituent of either $D^\lambda$ or $D^{\bar{\lambda}}$ for a partition $\lambda \vdash 14$ which can be obtained by adding a hook of length 1 to some partition $\mu \in M_{3,13}$. The dimensions of the ten relevant modules of $D^\lambda$ can be determined by using [28]. It turns out, in fact, that

$$\lambda \in M_{3,14} := \{(14), (13, 1), (12, 2), (12, 1^2)\}.$$

All of these restrict irreducibly to $\mathfrak{A}_{14}$. Similarly, for $n = 15$, we obtain $D = D^\lambda$ for

$$\lambda \in M_{3,15} := \{(15), (14, 1), (13, 2), (13, 1^2)\},$$

and so on. The assertion for $\ell = 3$ follows by induction.

The argument for $\ell \geqslant 5$ is similar. For $\ell = 0$, the degrees of the Specht modules are well known. □

### 3.3. *$\ell$-modular representations of* $2.\mathfrak{A}_n$

Here, we study low-dimensional faithful irreducible representations of the covering groups $2.\mathfrak{A}_n$. Since an irreducible 2-modular representation of $2.\mathfrak{A}_n$ is not faithful, we assume that $\ell \neq 2$ throughout this section. Also, we may and will assume that $n \geqslant 14$, since for smaller $n$ the modular tables are contained in [34] or in GAP.

We begin by looking at the covering groups of the symmetric groups. Their irreducible representations of characteristic 0 are indexed by partitions of $n$ into distinct parts, with the rule that one partition corresponds to two different representations if these have the same restriction to $2.\mathfrak{A}_n$. Following the notation in the paper of Wales [55], the representations of $2.\mathfrak{S}_n$ corresponding to a partition $\lambda$ are denoted by $<\lambda>$, or by $<\lambda>$ and $<\lambda>^*$ if there are two of them.

**Proposition 5.** *Let $n \geqslant 14$, and let $\ell \neq 13$ for $n = 14$. Then a faithful irreducible $F2.\mathfrak{S}_n$-module $M$ with $\dim M \leqslant 500$ is a constituent of the reduction modulo $\ell$ of $<n>$, or else of $<n>^*$.*

*Proof.* The assertion is correct for $n = 14$ and $\ell \neq 13$, and for $n = 15$ and $\ell = 13$. This follows from explicit computations with the character tables using MOC.

Suppose, then, that $n \geqslant 15$, and that $n \geqslant 16$ if $\ell = 13$. Let $M$ be a faithful simple $F2.\mathfrak{S}_{n+1}$-module with $\dim M \leqslant 500$. Then the restriction of $M$ to $2.\mathfrak{S}_n$ contains a faithful irreducible submodule of dimension at most 500. By induction, this is a composition factor of $< n >$, or else of $< n >^*$.

It follows that $M$ is a composition factor of the reduction modulo $\ell$ of $< n >^{2.\mathfrak{S}_{n+1}}$, or of $< n >^{*2.\mathfrak{S}_{n+1}}$. By the branching theorem, we have $< n >^{2.\mathfrak{S}_{n+1}} = < n+1 > + < n, 1 >$ if $n$ is odd, and $< n >^{2.\mathfrak{S}_{n+1}} = < n+1 > + < n+1 >^* + < n, 1 >$ if $n$ is even. By [55, Table IV], either a modular composition factor of $< n, 1 >$ is a composition factor of $< n+1 >$ or $< n+1 >^*$, or its dimension is larger than 500. This completes the proof. □

This proposition shows that for $n \geqslant 15$ and $n = 14$, $\ell \neq 13$, a faithful irreducible $F2.\mathfrak{A}_n$-module of dimension at most 250 is a composition factor of the restriction to $2.\mathfrak{A}_n$ of the reduction modulo $\ell$ of $< n >$ or of $< n >^*$. A direct computation with MOC shows that this is true even for $n = 14$ and $\ell = 13$. (Note that this is a cyclic defect case.) The modular composition factors of $< n >$ and $< n >^*$ are determined in [55]. In addition, it is also determined there which of them restrict irreducibly to $2.\mathfrak{A}_n$.

We thus obtain, from [55, Table III], the following corollary.

**Corollary 6.** *Let $n \geqslant 14$, let $\ell \neq 2$, and let $D$ be a faithful absolutely irreducible $2.\mathfrak{A}_n$-module of dimension $\dim(D) \leqslant 250$. Then $n \leqslant 18$ and $D$ is a composition factor of a basic spin representation of $2.\mathfrak{A}_n$. We have the following cases for $(n, \dim(D))$:*

(1) $(14, 32)$ *for $\ell = 7$ (two modules $D$);*

(2) $(14, 64)$ *for $\ell \neq 7$ (one $D$);*

(3) $(15, 64)$ *for $\ell = 3, 5$ (one $D$);*

(4) $(15, 64)$ *for $\ell \neq 3, 5$ (two modules $D$);*

(5) $(16, 128)$ *(one $D$);*

(6) $(17, 128)$ *for $\ell = 17$ (one $D$);*

(7) $(17, 128)$ *for $\ell \neq 17$ (two modules $D$);*

(8) $(18, 128)$ *for $\ell = 3$ (two modules $D$).*

## 4. *The sporadic groups*

Our first source was the Modular Atlas [34], which contains the modular character tables for ten of the 26 sporadic groups (and their covering groups). Apart from these ten groups, the largest of which is the McLaughlin group, the modular character tables are completely known for the six sporadic groups He, 2.Ru, 6.Suz, 3.O′N, $Co_3$ and $Co_2$. Modular character tables for some of the larger groups are also available in some characteristics, for example for the Fischer groups $6.Fi_{22}$ and $Fi_{23}$ in characteristic 5, or for the Conway group $2.Co_1$ in characteristic 7. Moreover, for primes dividing the group order only once, the character degrees can be computed from the Brauer trees given in [21].

There are several sources for these character tables, not all of which have as yet been published. The only published source on the Held group is Ryba's paper [47]. The 2-modular character table for the covering group of the Rudvalis group was determined by Wilson (unpublished); its 3-modular tables are contained in [19], and its 5-modular tables in [24]. On the Suzuki group, there is only one paper, by Jansen and Müller [31]. The relevant papers for the O'Nan group and its covering group are [32], by Jansen and Wilson, and [17], by Henke, Hiss and Müller. The 2-modular characters of the two smallest Conway groups have been computed by Suleiman and Wilson [51, 52], their 3-modular characters by Jansen in his PhD thesis [30], and the 5-modular characters of $Co_3$ by Müller [43].

The 5-modular character table for $Co_2$ is contained in [33], as well as the 7-modular table for $2.Co_1$. The 5-modular Brauer characters for the Fischer groups $6.Fi_{22}$ and $Fi_{23}$ have been computed by Hiss, Lux, and White in [22] and [25]. This completes the list of known modular tables for the sporadic groups and their covering groups. The MOC website [42] contains the corresponding decomposition matrices and Brauer character degrees.

The latter reference also contains a list of minimal degrees of non-trivial representations of sporadic groups and their covering groups (except for the double cover of the Baby Monster), compiled by Christoph Jansen. It follows from this list that we do not have to consider the two Fischer groups $Fi_{23}$ and $Fi_{24}'$, nor the Baby Monster, B, or the Monster, M. For example, the minimal degree of a non-trivial representation of $Fi_{23}$ is 253. Since $Fi_{23}$ is a subgroup of the double cover 2.B of the Baby Monster, the smallest degree of the latter group is well over 250.

The same list also shows that for the Lyons group, Ly, we have only to worry about representations in characteristic 5, and for the largest Janko group, $J_4$, only about characteristic 2. The case of the Lyons group was handled by Klaus Lux and Alex Ryba (in unpublished work). They showed, using the condensation techniques of the Meat-Axe (see [48] or [40]), that the only non-trivial representation in characteristic 5 of the Lyons group of degree smaller than 250 is the known one of degree 111, constructed by Meyer, Neutsch and Parker [41]. Lux and Ryba also settled the case of the Harada–Norton group in characteristic 5 using the same techniques, but this has been checked independently, using the methods described in Section 2. The minimal degree of a faithful representation of the Conway group $Co_1$ in odd characteristic equals 276. Hence for $2.Co_1$ and odd characteristics, we have only to consider blocks containing faithful characters. The diploma thesis of Hensing [18] gives enough information on the 5-modular decomposition numbers of $2.Co_1$ to show that our list is correct in this case. Unpublished work of Rosenboom settles the case of $Co_1$ in characteristic 2, but we have also checked this case independently. The case of the smallest Fischer group $Fi_{22}$ and its covering group has been handled by Christoph Jansen, in unpublished work.

We are thus left with the groups HN, Th, $2.Co_1$ and $J_4$, the latter only in characteristic 2, and the Conway group only in characteristic 3. To deal with these cases, we used the methods and the MOC system described in Section 2.

## 5. *The Frobenius–Schur indicators*

We comment briefly on the methods used to determine the Frobenius–Schur indicators of the characters occurring in the tables below. Let us begin with the definition. Suppose that $G$ is a finite group, $K$ a field, and $M$ a finitely generated, absolutely irreducible $KG$-module. Suppose first that $M$ is self-dual. Then $M$ carries a non-degenerate $G$-invariant bilinear form, which is symmetric or alternating. Moreover, this form is unique up to scalar multiplication. If the characteristic of $K$ is 2, and $M$ is not the trivial module, then the invariant form is alternating. If the characteristic of $K$ is odd and $M$ carries a non-degenerate symmetric bilinear form, this form is the polarization of a quadratic form on $M$, which is non-degenerate and $G$-invariant. If $M$ is not self-dual, it does not carry any non-trivial $G$-invariant bilinear form.

The Frobenius–Schur indicator of $M$ is an integer of the set $\{-1, 0, +1\}$. It is defined to be 0 if and only if $M$ is not self-dual. The Frobenius–Schur indicator of $M$ is set to be $+1$ if and only if $M$ carries a non-degenerate $G$-invariant quadratic form. Thus the indicator of $M$ is $-1$ if and only if $M$ is self-dual and carries a non-degenerate $G$-invariant bilinear alternating form, but no $G$-invariant quadratic form.

In our tables the indicators are represented by one of the three symbols $\{-, \circ, +\}$, with the obvious bijection.

To compute the Frobenius–Schur indicators, we used several well-known methods. We begin our discussion with the case where $K$ has odd characteristic $p$, and $M$ is self-dual. Then the Frobenius–Schur indicator of $M$ can be computed from the $p$-modular decomposition matrix of $G$ and the ordinary character table of $G$ as follows. There is a self-dual ordinary irreducible module $V$ of $G$ whose reduction modulo $p$ contains $M$ with odd multiplicity, and the Frobenius–Schur indicator of $M$ is equal to that of $V$. This result is due, independently, to Thompson [53] and Willems [59] (see the survey [60], in particular Theorem 2.8 therein).

In our case, the self-dual ordinary irreducible modules containing $M$ with odd multiplicity can be easily found; in fact, most of the time $M$ is liftable to a self-dual ordinary irreducible module. Thus it remains only to compute the Frobenius–Schur indicators of certain ordinary irreducible modules.

Let $V$ be an irreducible $\mathbb{C}G$-module with character $\chi$. Then the Frobenius–Schur indicator of $V$ is equal to

$$\nu_2(\chi) := \frac{1}{|G|} \sum_{g \in G} \chi(g^2).$$

Thus the indicators in characteristic 0 can be computed from the ordinary character table and the 2-power map of $G$. Also, $\nu_2(\chi) = +1$ if and only if $V \cong \mathbb{C} \otimes_{\mathbb{R}} V_0$ for some $\mathbb{R}G$-module $V_0$; in other words, if and only if $\chi$ is afforded by a real representation of $G$ (see [26, Chapter 4]). If the character table of $G$ is not available, we use the latter criterion.

Let $\chi$ be real-valued. To decide whether or not $\chi$ is afforded by a real representation, we use the real Schur index $m_{\mathbb{R}}(\chi)$ (see [26, Definition 10.1]). By the definition of the Schur index and the discussion above, we have $\nu_2(\chi) = -1$ if and only if $m_{\mathbb{R}}(\chi) \neq 1$. Since $|\mathbb{C} : \mathbb{R}| = 2$, the latter condition is equivalent to $m_{\mathbb{R}}(\chi) = 2$, and implies that $\chi(1)$ is even (see [26, Corollary 10.2(g) and (h)]). Suppose that $\psi$ is the character of a real representation of $G$. Then, by [26, Corollary 10.2(c)], the Schur index $m_{\mathbb{R}}(\chi)$ divides the scalar product $(\psi, \chi)$. This can be applied in particular in cases where $\psi$ is induced or restricted from a character of a real representation of a subgroup or overgroup of $G$.

Results on Schur indices of characters of groups of Lie type can be found in the papers [10, 11] by Rod Gow. We also made use of the results of Przygocki [46], on the Schur indices of the symplectic groups, and of Tiep [54, Corollary 4.5], on the Schur indices of the Weil representations of the unitary groups.

Let us suppose now that the characteristic of $K$ equals 2. Here, no simple method is known for computing the Frobenius–Schur indicators. A collection of various methods is contained in the articles [13, 14] of Gow and Willems. We made use of [13, Lemmas 1.1 and 1.2] in particular, and of [14, Theorem 1.1 and its proof].

If the $KG$-module $M$ lifts to an absolutely irreducible $\mathbb{Q}G$-module, then the Frobenius–Schur indicator of $M$ equals $+1$. This is a well-known, elementary result, which is included in the much more general [13, Theorem 2.2].

To determine the indicators of the basic spin representations of the alternating groups, we followed a suggestion of Rod Gow. First, by [12, Corollary 4.3], the spin module of the symplectic group $S_{2m}(q)$ (where $q$ is even) of degree $2^m$ has indicator $+1$, if $m \geqslant 3$ (see also [35, Proposition 5.4.9]). The restriction of the spin module of $S_{2m}(q)$ to the subgroup $\mathfrak{S}_{2m+1}$ or $\mathfrak{S}_{2m+2}$ remains irreducible. Thus the basic spin representation of the symmetric group $\mathfrak{S}_n$ has indicator $+1$ if $n \geqslant 7$. This implies, using for example [13, Lemma 1.2],

Table 1: Some Frobenius–Schur indicators

| $d$ | $G$ | $\ell$ | ind |
|---|---|---|---|
| 32 | $2.\mathfrak{A}_{14}$ | 7 | $-$ |
| 104 | $\mathrm{U}_4(5)$ | 2 | $-$ |
| 174 | $\mathrm{S}_4(7)$ | 2 | $+$ |
| 208 | $\mathfrak{A}_{14}$ | 2 | $+$ |
| 218 | ${}^3D_4(3)$ | 2 | $+$ |

that the basic spin representations for $\mathfrak{A}_n$ also have indicator $+1$, provided that they are self-dual. The splitting of the basic spin representation of $\mathfrak{S}_n$ on restriction to $\mathfrak{A}_n$ has been determined by Benson [**2**, Theorems 1.1 and 6.1].

The Frobenius–Schur indicators for the sporadic groups have mainly been determined by Richard Parker and Rob Wilson, using the computational methods described in [**52**]. In addition, Rob Wilson computed the indicators of the representations shown in Table 1, which remained open in a previous version of this paper.

## 6. *A table of low-dimensional absolutely irreducible representations of quasi-simple groups*

In order to shorten the table of low-dimensional representations, we have grouped some generic cases together in Table 2. The corresponding entries have been omitted from Table 3 for $\mathfrak{A}_n$ with $n \geqslant 14$, and for $\mathrm{L}_2(q)$ for $q \neq 9$. The indicators for $\mathrm{L}_2(q)$ when $\ell = 2$ were taken from [**13**, Theorem 2.3] for the characters of degree $q \pm 1$, and from [**14**, Theorem 1.2] for the Weil character of degree $(q-1)/2$.

It may be helpful to point out that, in accordance with the formulation of Theorem 1, the following pairs $(G/Z(G), \ell)$ have been omitted from the tables:

$\mathfrak{A}_6 \cong \mathrm{S}_4(2)' \cong \mathrm{L}_2(9)$ in characteristic $\ell = 3$,
$\mathfrak{A}_8 \cong \mathrm{L}_4(2)$ in characteristic $\ell = 2$,
$\mathrm{L}_2(7) \cong \mathrm{L}_3(2)$ in characteristic $\ell = 2, 7$,
$\mathrm{U}_4(2) \cong \mathrm{S}_4(3)$ in characteristic $\ell = 2, 3$,
$G_2(2)' \cong \mathrm{U}_3(3)$ in characteristic $\ell = 3$,
${}^2G_2(3)' \cong \mathrm{L}_2(8)$ in characteristic $\ell = 2$.

The Tits group, that is, the derived group ${}^2F_4(2)'$ of the Ree group of type $F_4$ over $\mathbb{F}_2$, cannot be obtained as a factor group of the group of fixed points under a Frobenius morphism of an algebraic group, so it is not considered as a group of Lie type in characteristic 2 here.

Let us briefly explain the notation used in the tables. The columns headed '$d$' contain the degrees. The columns headed '$\ell$' specify the characteristics of the fields over which the respective representations are defined. An entry '$\neq \ell$' for a prime $\ell$ indicates that a representation of this degree exists for all other characteristics than $\ell$, including 0. The 'field' columns give the irrationalities of the Brauer characters (using Atlas notation, except for the irrationalities occurring in Table 2). Finally, the last column gives the Frobenius–Schur indicators, which are defined in the section above.

Table 2: Generic examples

(a) Alternating groups

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| $n-2$ | $\mathfrak{A}_n$ | $\ell \mid n$ | | $+$ |
| $n-1$ | $\mathfrak{A}_n$ | $\ell \nmid n$ | | $+$ |

(b) $L_2(q),\ q \equiv 1 \pmod 4,\ \ell \nmid q$

| $d$ | $G$ | condition on $\ell$ | field | ind |
|---|---|---|---|---|
| $(q-1)/2$ | $L_2(q)$ | 2 | $\sqrt{q}$ | $-$ |
| $(q-1)/2$ | $2.L_2(q)$ | $\ell \neq 2$ | $\sqrt{q}$ | $-$ |
| $(q+1)/2$ | $L_2(q)$ | $\ell \neq 2$ | $\sqrt{q}$ | $+$ |
| $q-1$ | $L_2(q)$ | | $\zeta_{q+1}^j + \zeta_{q+1}^{-j}$ | $+$ |
| $q-1$ | $2.L_2(q)$ | $((q+1)/2)_{\ell'} \neq 1$ | $\zeta_{q+1}^j + \zeta_{q+1}^{-j}$ | $-$ |
| $q$ | $L_2(q)$ | $\ell \nmid (q+1)$ | | $+$ |
| $q+1$ | $L_2(q)$ | $((q-1)/4)_{\ell'} \neq 1$ | $\zeta_{q-1}^{2j} + \zeta_{q-1}^{-2j}$ | $+$ |
| $q+1$ | $2.L_2(q)$ | $\ell \neq 2$ | $\zeta_{q-1}^j + \zeta_{q-1}^{-j}$ | $-$ |

(c) $L_2(q),\ q \equiv 3 \pmod 4,\ \ell \nmid q$

| $d$ | $G$ | condition on $\ell$ | field | ind |
|---|---|---|---|---|
| $(q-1)/2$ | $L_2(q)$ | | $\sqrt{-q}$ | $\circ$ |
| $(q+1)/2$ | $2.L_2(q)$ | $\ell \neq 2$ | $\sqrt{-q}$ | $\circ$ |
| $q-1$ | $L_2(q)$ | | $\zeta_{q+1}^{2j} + \zeta_{q+1}^{-2j}$ | $+$ |
| $q-1$ | $2.L_2(q)$ | $\ell \neq 2$ | $\zeta_{q+1}^j + \zeta_{q+1}^{-j}$ | $-$ |
| $q$ | $L_2(q)$ | $\ell \nmid (q+1)$ | | $+$ |
| $q+1$ | $L_2(q)$ | $((q-1)/2)_{\ell'} \neq 1$ | $\zeta_{q-1}^j + \zeta_{q-1}^{-j}$ | $+$ |
| $q+1$ | $2.L_2(q)$ | $((q-1)/2)_{\ell'} \neq 1, \ell \neq 2$ | $\zeta_{q-1}^j + \zeta_{q-1}^{-j}$ | $-$ |

(d) $L_2(q),\ q \equiv 0 \pmod 2,\ \ell \nmid q$

| $d$ | $G$ | condition on $\ell$ | field | ind |
|---|---|---|---|---|
| $q-1$ | $L_2(q)$ | | $\zeta_{q+1}^j + \zeta_{q+1}^{-j}$ | $+$ |
| $q$ | $L_2(q)$ | $\ell \nmid (q+1)$ | | $+$ |
| $q+1$ | $L_2(q)$ | $(q-1)_{\ell'} \neq 1$ | $\zeta_{q-1}^j + \zeta_{q-1}^{-j}$ | $+$ |

Table 3: Absolutely irreducible representations of quasi-simple groups

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 3 | $3.\mathfrak{A}_6$ | $0, 2$ | $z3, b5$ | $\circ$ |
| 3 | $3.\mathfrak{A}_6$ | 5 | $z3$ | $\circ$ |
| 3 | $3.\mathfrak{A}_7$ | 5 | $z3, b7$ | $\circ$ |
| 4 | $\mathfrak{A}_6$ | 2 | | $-$ |
| 4 | $2.\mathfrak{A}_6$ | $0, 5$ | | $-$ |
| 4 | $\mathfrak{A}_7$ | 2 | $b7$ | $\circ$ |
| 4 | $2.\mathfrak{A}_7$ | 7 | | $-$ |
| 4 | $2.\mathfrak{A}_7$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 4 | $4_2.\mathrm{L}_3(4)$ | 3 | $i1, r7$ | $\circ$ |
| 4 | $2.\mathrm{U}_4(2)$ | $0, 5$ | $z3$ | $\circ$ |
| 5 | $\mathfrak{A}_6$ | $0, 5$ | | $+$ |
| 5 | $\mathfrak{A}_7$ | 7 | | $+$ |
| 5 | $\mathrm{U}_4(2)$ | $0, 5$ | $z3$ | $\circ$ |
| 5 | $\mathrm{M}_{11}$ | 3 | $i2, b11$ | $\circ$ |
| 6 | $3.\mathfrak{A}_6$ | $0, 5$ | $z3$ | $\circ$ |
| 6 | $6.\mathfrak{A}_6$ | $0, 5$ | $z3, r2$ | $\circ$ |
| 6 | $\mathfrak{A}_7$ | $\neq 7$ | | $+$ |
| 6 | $2.\mathfrak{A}_7$ | 3 | $r2$ | $-$ |
| 6 | $3.\mathfrak{A}_7$ | $\neq 3$ | $z3$ | $\circ$ |
| 6 | $6.\mathfrak{A}_7$ | $\neq 2, 3$ | $z3, r2$ | $\circ$ |
| 6 | $2.\mathrm{L}_3(4)$ | 3 | | $+$ |
| 6 | $6.\mathrm{L}_3(4)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 6 | $\mathrm{U}_3(3)$ | $\neq 3$ | | $-$ |
| 6 | $\mathrm{U}_4(2)$ | $0, 5$ | | $+$ |
| 6 | $3_1.\mathrm{U}_4(3)$ | 2 | $z3$ | $\circ$ |
| 6 | $6_1.\mathrm{U}_4(3)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 6 | $2.\mathrm{M}_{12}$ | 3 | $i2, i5, b11$ | $\circ$ |
| 6 | $3.\mathrm{M}_{22}$ | 2 | $z3, b11$ | $\circ$ |
| 6 | $\mathrm{J}_2$ | 2 | $b5$ | $-$ |
| 6 | $2.\mathrm{J}_2$ | 5 | | $-$ |
| 6 | $2.\mathrm{J}_2$ | $\neq 2, 5$ | $b5$ | $-$ |
| 7 | $\mathfrak{A}_8$ | $\neq 2$ | | $+$ |
| 7 | $\mathfrak{A}_9$ | 3 | | $+$ |
| 7 | $\mathrm{U}_3(3)$ | $0, 7$ | | $+$ |
| 7 | $\mathrm{U}_3(3)$ | $0, 7$ | $i1$ | $\circ$ |
| 7 | $\mathrm{S}_6(2)$ | $\neq 2$ | | $+$ |
| 7 | $\mathrm{J}_1$ | 11 | $b5, c19$ | $+$ |
| 8 | $\mathfrak{A}_6$ | $0, 2$ | $b5$ | $+$ |
| 8 | $\mathfrak{A}_6$ | 5 | | $+$ |
| 8 | $2.\mathfrak{A}_6$ | 0 | $b5$ | $-$ |
| 8 | $\mathfrak{A}_7$ | 5 | | $+$ |
| 8 | $2.\mathfrak{A}_8$ | $\neq 2$ | | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 8 | $\mathfrak{A}_9$ | $\neq 3$ | | $+$ |
| 8 | $2.\mathfrak{A}_9$ | $\neq 2$ | | $+$ |
| 8 | $\mathfrak{A}_{10}$ | 2 | | $-$ |
| 8 | $\mathfrak{A}_{10}$ | 5 | | $+$ |
| 8 | $2.\mathfrak{A}_{10}$ | 5 | $r6, r21$ | $+$ |
| 8 | $4_1.\mathrm{L}_3(4)$ | 5 | $i1$ | $\circ$ |
| 8 | $4_1.\mathrm{L}_3(4)$ | $\neq 2, 5$ | $i1, b5$ | $\circ$ |
| 8 | $2.\mathrm{S}_6(2)$ | $\neq 2$ | | $+$ |
| 8 | $2.\mathrm{O}_8^+(2)$ | $\neq 2$ | | $+$ |
| 8 | $2.\mathrm{Sz}(8)$ | 5 | $c_{13}$ | $+$ |
| 9 | $\mathfrak{A}_6$ | 0 | | $+$ |
| 9 | $3.\mathfrak{A}_6$ | $0, 2$ | $z3$ | $\circ$ |
| 9 | $3.\mathfrak{A}_7$ | 7 | $z3$ | $\circ$ |
| 9 | $\mathfrak{A}_{10}$ | $\neq 2, 5$ | | $+$ |
| 9 | $\mathfrak{A}_{11}$ | 11 | | $+$ |
| 9 | $\mathrm{M}_{11}$ | 11 | | $+$ |
| 9 | $3.\mathrm{J}_3$ | 2 | $z3, b17, b19$ | $\circ$ |
| 10 | $\mathfrak{A}_6$ | $0, 5$ | | $+$ |
| 10 | $2.\mathfrak{A}_6$ | $0, 5$ | $r2$ | $-$ |
| 10 | $\mathfrak{A}_7$ | 7 | | $+$ |
| 10 | $\mathfrak{A}_7$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 10 | $\mathfrak{A}_{11}$ | $\neq 11$ | | $+$ |
| 10 | $\mathfrak{A}_{12}$ | $2, 3$ | | $+$ |
| 10 | $2.\mathrm{L}_3(4)$ | 7 | | $+$ |
| 10 | $2.\mathrm{L}_3(4)$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 10 | $\mathrm{U}_4(2)$ | $0, 5$ | $z3$ | $\circ$ |
| 10 | $\mathrm{U}_5(2)$ | $\neq 2$ | | $-$ |
| 10 | $\mathrm{M}_{11}$ | $\neq 11$ | | $+$ |
| 10 | $\mathrm{M}_{11}$ | $\neq 2$ | $i2$ | $\circ$ |
| 10 | $\mathrm{M}_{12}$ | $2, 3$ | | $+$ |
| 10 | $2.\mathrm{M}_{12}$ | $\neq 2$ | $i2$ | $\circ$ |
| 10 | $\mathrm{M}_{22}$ | 2 | $b7$ | $\circ$ |
| 10 | $2.\mathrm{M}_{22}$ | 7 | | $+$ |
| 10 | $2.\mathrm{M}_{22}$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 11 | $\mathfrak{A}_{12}$ | $\neq 2, 3$ | | $+$ |
| 11 | $\mathfrak{A}_{13}$ | 13 | | $+$ |
| 11 | $\mathrm{U}_5(2)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 11 | $\mathrm{M}_{11}$ | $\neq 2, 3$ | | $+$ |
| 11 | $\mathrm{M}_{12}$ | $\neq 2, 3$ | | $+$ |
| 11 | $\mathrm{M}_{23}$ | 2 | $b7, i15, b23$ | $\circ$ |
| 11 | $\mathrm{M}_{24}$ | 2 | $b7, i15, b23$ | $\circ$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 12 | $6.\mathfrak{A}_6$ | 0 | $z3, b5$ | $\circ$ |
| 12 | $6.\mathfrak{A}_7$ | 5 | $z3, b7$ | $\circ$ |
| 12 | $\mathfrak{A}_{13}$ | $\neq 13$ | | $+$ |
| 12 | $12_2.\mathrm{L}_3(4)$ | 7 | $z12, b5$ | $\circ$ |
| 12 | $\mathrm{U}_3(4)$ | $\neq 2$ | | $-$ |
| 12 | $\mathrm{S}_4(5)$ | 2 | $b5$ | $-$ |
| 12 | $2.\mathrm{S}_4(5)$ | $\neq 2, 5$ | $b5$ | $-$ |
| 12 | $2.G_2(4)$ | $\neq 2$ | | $-$ |
| 12 | $2.\mathrm{M}_{12}$ | $\neq 2, 3$ | | $+$ |
| 12 | 2.Suz | 3 | | $-$ |
| 12 | 3.Suz | 2 | $z3$ | $\circ$ |
| 12 | 6.Suz | $\neq 2, 3$ | $z3$ | $\circ$ |
| 13 | $\mathfrak{A}_7$ | 3, 5 | | $+$ |
| 13 | $\mathfrak{A}_8$ | 3, 5 | | $+$ |
| 13 | $\mathrm{U}_3(4)$ | $\neq 2, 5$ | $z5$ | $\circ$ |
| 13 | $\mathrm{S}_4(5)$ | $\neq 2, 5$ | $b5$ | $+$ |
| 13 | $\mathrm{S}_6(3)$ | $\neq 3$ | $z3$ | $\circ$ |
| 13 | $\mathrm{J}_2$ | 3 | $b5$ | $+$ |
| 14 | $\mathfrak{A}_7$ | $\neq 3, 5$ | | $+$ |
| 14 | $2.\mathfrak{A}_7$ | $\neq 2, 3$ | $r2$ | $-$ |
| 14 | $\mathfrak{A}_8$ | 0, 7 | | $+$ |
| 14 | $\mathrm{U}_3(3)$ | $\neq 3$ | | $+$ |
| 14 | $\mathrm{S}_6(2)$ | 3 | | $+$ |
| 14 | $2.\mathrm{S}_6(3)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 14 | Sz(8) | $\neq 2$ | $i1$ | $\circ$ |
| 14 | $G_2(3)$ | $\neq 3$ | | $+$ |
| 14 | $\mathrm{J}_1$ | 11 | $b5, c19$ | $+$ |
| 14 | $\mathrm{J}_2$ | 5 | | $+$ |
| 14 | $\mathrm{J}_2$ | $\neq 3, 5$ | $b5$ | $+$ |
| 14 | $2.\mathrm{J}_2$ | $\neq 2$ | | $-$ |
| 15 | $3.\mathfrak{A}_6$ | 0, 5 | $z3$ | $\circ$ |
| 15 | $\mathfrak{A}_7$ | $\neq 2, 7$ | | $+$ |
| 15 | $3.\mathfrak{A}_7$ | $\neq 3$ | $z3$ | $\circ$ |
| 15 | $\mathrm{L}_3(4)$ | 3 | | $+$ |
| 15 | $3.\mathrm{L}_3(4)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 15 | $\mathrm{U}_4(2)$ | 0, 5 | | $+$ |
| 15 | $3_1.\mathrm{U}_4(3)$ | $\neq 3$ | $z3$ | $\circ$ |
| 15 | $\mathrm{S}_6(2)$ | $\neq 2, 3$ | | $+$ |
| 15 | $\mathrm{M}_{12}$ | 3 | $b11$ | $\circ$ |
| 15 | $3.\mathrm{M}_{22}$ | 2 | $z3, b11$ | $\circ$ |
| 16 | $2.\mathfrak{A}_7$ | 7 | | $-$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 16 | $2.\mathfrak{A}_8$ | 7 | | $-$ |
| 16 | $\mathfrak{A}_{10}$ | 2 | | $+$ |
| 16 | $2.\mathfrak{A}_{10}$ | $\neq 2, 5$ | | $+$ |
| 16 | $\mathfrak{A}_{11}$ | 2 | $b11$ | $\circ$ |
| 16 | $2.\mathfrak{A}_{11}$ | 11 | | $+$ |
| 16 | $2.\mathfrak{A}_{11}$ | $\neq 2, 11$ | $b11$ | $\circ$ |
| 16 | $\mathfrak{A}_{12}$ | 2 | $z3$ | $\circ$ |
| 16 | $2.\mathfrak{A}_{12}$ | 3 | $i2, i5, r7, b11$ | $\circ$ |
| 16 | $4_2.\mathrm{L}_3(4)$ | 3 | $i1, r7$ | $\circ$ |
| 16 | $2.\mathrm{Sz}(8)$ | 13 | $y7$ | $+$ |
| 16 | $\mathrm{M}_{11}$ | 11 | | $+$ |
| 16 | $\mathrm{M}_{11}$ | $\neq 3, 11$ | $b11$ | $\circ$ |
| 16 | $\mathrm{M}_{12}$ | 11 | | $+$ |
| 16 | $\mathrm{M}_{12}$ | $\neq 3, 11$ | $b11$ | $\circ$ |
| 16 | $4.\mathrm{M}_{22}$ | 7 | $i1, r11$ | $\circ$ |
| 18 | $3.\mathfrak{A}_7$ | 5 | $z3, b7$ | $\circ$ |
| 18 | $\mathrm{S}_4(4)$ | $\neq 2$ | | $+$ |
| 18 | $\mathrm{J}_3$ | 3 | $b5$ | $+$ |
| 18 | $3.\mathrm{J}_3$ | 5 | $z3$ | $\circ$ |
| 18 | $3.\mathrm{J}_3$ | $\neq 3, 5$ | $z3, b5$ | $\circ$ |
| 19 | $\mathfrak{A}_8$ | 7 | | $+$ |
| 19 | $\mathfrak{A}_9$ | 7 | | $+$ |
| 19 | $\mathrm{L}_3(4)$ | 3, 7 | | $+$ |
| 20 | $\mathfrak{A}_7$ | 2 | | $-$ |
| 20 | $2.\mathfrak{A}_7$ | $\neq 2, 3$ | | $-$ |
| 20 | $\mathfrak{A}_8$ | 0, 5 | | $+$ |
| 20 | $\mathfrak{A}_9$ | 2 | $i15$ | $\circ$ |
| 20 | $\mathrm{L}_3(4)$ | 0, 5 | | $+$ |
| 20 | $4_2.\mathrm{L}_3(4)$ | $\neq 2, 3$ | $i1$ | $\circ$ |
| 20 | $\mathrm{U}_3(5)$ | $\neq 5$ | | $-$ |
| 20 | $\mathrm{U}_4(2)$ | 0, 5 | | $+$ |
| 20 | $2.\mathrm{U}_4(2)$ | 0, 5 | | $-$ |
| 20 | $2.\mathrm{U}_4(2)$ | 0, 5 | $z3$ | $\circ$ |
| 20 | $\mathrm{U}_4(3)$ | 2 | | $+$ |
| 20 | $2.\mathrm{U}_4(3)$ | $\neq 2, 3$ | | $-$ |
| 20 | $4.\mathrm{U}_4(3)$ | $\neq 2, 3$ | $i1$ | $\circ$ |
| 20 | $\mathrm{M}_{22}$ | 11 | | $+$ |
| 20 | $\mathrm{J}_1$ | 2 | | $+$ |
| 20 | HS | 2 | | $-$ |
| 21 | $\mathfrak{A}_7$ | 0, 7 | | $+$ |
| 21 | $3.\mathfrak{A}_7$ | $\neq 2, 3$ | $z3$ | $\circ$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 21 | $\mathfrak{A}_8$ | 0, 7 | $i15$ | ∘ |
| 21 | $\mathfrak{A}_8$ | $\neq 2$ | | + |
| 21 | $\mathfrak{A}_9$ | 0, 7 | $i15$ | ∘ |
| 21 | $\mathfrak{A}_9$ | 3, 5 | | + |
| 21 | $3.L_3(4)$ | $\neq 2, 3$ | $z3$ | ∘ |
| 21 | $U_3(3)$ | 0, 7 | | + |
| 21 | $U_3(3)$ | 0, 7 | $i1$ | ∘ |
| 21 | $U_3(5)$ | $\neq 2, 5$ | | + |
| 21 | $3.U_3(5)$ | $\neq 3, 5$ | $z3$ | ∘ |
| 21 | $U_4(3)$ | $\neq 2, 3$ | | + |
| 21 | $3_1.U_4(3)$ | $\neq 2, 3$ | $z3$ | ∘ |
| 21 | $U_6(2)$ | 3 | | + |
| 21 | $3.U_6(2)$ | $\neq 2, 3$ | $z3$ | ∘ |
| 21 | $S_6(2)$ | $\neq 2$ | | + |
| 21 | $M_{22}$ | $\neq 2, 11$ | | + |
| 21 | $3.M_{22}$ | $\neq 2, 3$ | $z3$ | ∘ |
| 21 | $M_{23}$ | 23 | | + |
| 21 | $J_2$ | 5 | | + |
| 21 | $J_2$ | $\neq 2, 5$ | $b5$ | + |
| 21 | HS | 5 | | + |
| 21 | McL | 3, 5 | | + |
| 22 | $2.L_3(4)$ | 3 | $b5$ | + |
| 22 | $U_6(2)$ | $\neq 2, 3$ | | + |
| 22 | $M_{23}$ | $\neq 2, 23$ | | + |
| 22 | $M_{24}$ | 3 | | + |
| 22 | $J_1$ | 19 | $b5$ | + |
| 22 | HS | $\neq 2, 5$ | | + |
| 22 | McL | $\neq 3, 5$ | | + |
| 22 | $Co_3$ | 2 | | − |
| 22 | $Co_3$ | 3 | | + |
| 22 | $Co_2$ | 2 | | + |
| 23 | $U_4(2)$ | 5 | | + |
| 23 | $M_{24}$ | $\neq 2, 3$ | | + |
| 23 | $Co_3$ | $\neq 2, 3$ | | + |
| 23 | $Co_2$ | $\neq 2$ | | + |
| 24 | $3.\mathfrak{A}_7$ | 0, 2 | $z3$, $b7$ | ∘ |
| 24 | $6.\mathfrak{A}_7$ | 7 | $z3$ | ∘ |
| 24 | $6.\mathfrak{A}_7$ | 0, 5 | $z3$, $b7$ | ∘ |
| 24 | $2.\mathfrak{A}_8$ | $\neq 2, 7$ | $b7$ | ∘ |
| 24 | $4_1.L_3(4)$ | 3 | $i1$, $r7$ | ∘ |
| 24 | $12_1.L_3(4)$ | 7 | $z12$ | ∘ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 24 | $12_1.\mathrm{L}_3(4)$ | $0, 5$ | $z12, b7$ | $\circ$ |
| 24 | $\mathrm{U}_4(2)$ | $0$ | | $+$ |
| 24 | $\mathrm{S}_4(7)$ | $2$ | $b7$ | $\circ$ |
| 24 | $2.\mathrm{S}_4(7)$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 24 | $2.\mathrm{Sz}(8)$ | $13$ | $y7$ | $+$ |
| 24 | $\mathrm{M}_{11}$ | $3$ | | $+$ |
| 24 | $12.\mathrm{M}_{22}$ | $11$ | $z24, b7$ | $\circ$ |
| 24 | $\mathrm{Co}_1$ | $2$ | | $+$ |
| 24 | $2.\mathrm{Co}_1$ | $\neq 2$ | | $+$ |
| 25 | $\mathrm{S}_4(7)$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 25 | ${}^3D_4(2)$ | $3$ | | $+$ |
| 26 | $\mathfrak{A}_9$ | $2$ | | $+$ |
| 26 | $\mathfrak{A}_{10}$ | $2$ | | $+$ |
| 26 | $2.\mathrm{L}_3(4)$ | $7$ | | $+$ |
| 26 | $\mathrm{L}_4(3)$ | $\neq 3$ | | $+$ |
| 26 | $\mathrm{U}_3(3)$ | $7$ | | $+$ |
| 26 | $\mathrm{S}_6(2)$ | $7$ | | $+$ |
| 26 | ${}^3D_4(2)$ | $\neq 2, 3$ | | $+$ |
| 26 | ${}^2F_4(2)'$ | $2$ | | $+$ |
| 26 | ${}^2F_4(2)'$ | $\neq 2$ | $i2$ | $\circ$ |
| 27 | $\mathfrak{A}_9$ | $\neq 2, 7$ | | $+$ |
| 27 | $\mathrm{U}_3(3)$ | $0$ | | $+$ |
| 27 | $\mathrm{S}_6(2)$ | $\neq 2, 7$ | | $+$ |
| 27 | $3.\mathrm{O}_7(3)$ | $\neq 3$ | $z3$ | $\circ$ |
| 27 | $3.G_2(3)$ | $\neq 3$ | $z3$ | $\circ$ |
| 27 | ${}^2F_4(2)'$ | $\neq 2$ | $i1$ | $\circ$ |
| 27 | $\mathrm{J}_1$ | $11$ | $b5, c19$ | $+$ |
| 27 | $3.\mathrm{Fi}_{22}$ | $2$ | $z3, b11$ | $\circ$ |
| 28 | $\mathfrak{A}_8$ | $\neq 2, 5$ | | $+$ |
| 28 | $\mathfrak{A}_9$ | $\neq 2, 3$ | | $+$ |
| 28 | $\mathfrak{A}_{10}$ | $5$ | | $+$ |
| 28 | $2.\mathrm{L}_3(4)$ | $5$ | | $+$ |
| 28 | $2.\mathrm{L}_3(4)$ | $0, 7$ | $b5$ | $+$ |
| 28 | $4_2.\mathrm{L}_3(4)$ | $5$ | $i1$ | $\circ$ |
| 28 | $4_2.\mathrm{L}_3(4)$ | $\neq 2, 5$ | $i1, b5$ | $\circ$ |
| 28 | $\mathrm{U}_3(3)$ | $0, 7$ | $i1$ | $\circ$ |
| 28 | $\mathrm{U}_3(5)$ | $\neq 5$ | | $+$ |
| 28 | $\mathrm{O}_8^+(2)$ | $\neq 2$ | | $+$ |
| 28 | $2.\mathrm{M}_{22}$ | $5$ | $i1, r11$ | $\circ$ |
| 28 | $2.\mathrm{HS}$ | $5$ | $i1, r11$ | $\circ$ |
| 28 | $\mathrm{Ru}$ | $2$ | | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

Continued from the previous page

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 28 | 2.Ru | $\neq 2$ | $i1$ | $\circ$ |
| 29 | $L_3(5)$ | 31 | | $+$ |
| 29 | $L_5(2)$ | 31 | | $+$ |
| 29 | $M_{12}$ | 11 | | $+$ |
| 30 | $L_3(5)$ | $\neq 5, 31$ | | $+$ |
| 30 | $L_5(2)$ | $\neq 2, 31$ | | $+$ |
| 30 | $U_4(2)$ | 0, 5 | | $+$ |
| 30 | $U_4(2)$ | 0, 5 | $z3$ | $\circ$ |
| 31 | $L_3(5)$ | $\neq 2, 5$ | | $+$ |
| 31 | $L_3(5)$ | $\neq 2, 5$ | $i1$ | $\circ$ |
| 31 | $J_1$ | 7 | $c19$ | $+$ |
| 32 | $2.\mathfrak{A}_8$ | 5 | $z3$, $b7$ | $\circ$ |
| 32 | $2.\mathfrak{A}_{12}$ | $\neq 2, 3$ | | $-$ |
| 32 | $\mathfrak{A}_{13}$ | 2 | $b13$ | $+$ |
| 32 | $2.\mathfrak{A}_{13}$ | 13 | | $-$ |
| 32 | $2.\mathfrak{A}_{13}$ | $\neq 2, 13$ | $b13$ | $-$ |
| 32 | $2.\mathfrak{A}_{14}$ | 7 | $r3$, $r6$, $r10$, $b5$, $b13$, $b33$ | $-$ |
| 32 | $U_3(3)$ | 0, 2 | $b7$ | $\circ$ |
| 32 | $2.U_4(2)$ | 5 | $z3$ | $\circ$ |
| 32 | $2.M_{12}$ | $\neq 2, 3$ | | $-$ |
| 33 | $S_4(4)$ | 5 | | $+$ |
| 33 | $O_8^-(2)$ | 7 | | $+$ |
| 34 | $\mathfrak{A}_9$ | 5 | | $+$ |
| 34 | $\mathfrak{A}_{10}$ | 3, 5 | | $+$ |
| 34 | $\mathfrak{A}_{11}$ | 3 | | $+$ |
| 34 | $U_4(3)$ | 2 | | $-$ |
| 34 | $S_4(4)$ | $\neq 2, 5$ | | $+$ |
| 34 | $S_6(2)$ | 3 | | $+$ |
| 34 | $O_8^-(2)$ | $\neq 2, 7$ | | $+$ |
| 34 | $M_{12}$ | 3 | | $+$ |
| 34 | $M_{22}$ | 2 | | $-$ |
| 34 | $J_1$ | 19 | $b5$ | $+$ |
| 35 | $\mathfrak{A}_7$ | $\neq 2, 3$ | | $+$ |
| 35 | $\mathfrak{A}_8$ | $\neq 2$ | | $+$ |
| 35 | $\mathfrak{A}_9$ | $\neq 2$ | | $+$ |
| 35 | $\mathfrak{A}_{10}$ | $\neq 2, 3$ | | $+$ |
| 35 | $\mathfrak{A}_{10}$ | 5 | $r21$ | $+$ |
| 35 | $L_3(4)$ | $\neq 2, 3$ | | $+$ |
| 35 | $U_4(3)$ | $\neq 2, 3$ | | $+$ |
| 35 | $S_6(2)$ | $\neq 2$ | | $+$ |
| 35 | $S_8(2)$ | $\neq 2$ | | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 35 | $O_8^+(2)$ | $\neq 2$ | | $+$ |
| 35 | Sz(8) | 13 | | $+$ |
| 35 | Sz(8) | $\neq 2, 13$ | $c_{13}$ | $+$ |
| 36 | $2.\mathfrak{A}_7$ | 0, 3 | | $-$ |
| 36 | $6.\mathfrak{A}_7$ | 0 | $z3$ | $\circ$ |
| 36 | $\mathfrak{A}_{10}$ | $\neq 2, 5$ | | $+$ |
| 36 | $\mathfrak{A}_{11}$ | 11 | | $+$ |
| 36 | $2.L_3(4)$ | $\neq 2, 7$ | | $+$ |
| 36 | $4_2.L_3(4)$ | $\neq 2$ | $i1$ | $\circ$ |
| 36 | $6.L_3(4)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 36 | $12_2.L_3(4)$ | $\neq 2, 3$ | $z12$ | $\circ$ |
| 36 | $2.U_4(2)$ | 0 | $z3$ | $\circ$ |
| 36 | $3_2.U_4(3)$ | $\neq 3$ | $z3$ | $\circ$ |
| 36 | $12_2.U_4(3)$ | $\neq 2, 3$ | $z12$ | $\circ$ |
| 36 | $6.M_{22}$ | 11 | $z12$ | $\circ$ |
| 36 | $J_2$ | $\neq 5$ | | $+$ |
| 36 | $2.J_2$ | 3 | $i1$ | $\circ$ |
| 38 | $L_4(3)$ | 2, 5 | | $+$ |
| 39 | $L_4(3)$ | 0, 13 | | $+$ |
| 39 | $U_3(4)$ | 5 | | $+$ |
| 39 | $U_3(4)$ | $\neq 2, 5$ | $b5$ | $+$ |
| 40 | $2.L_4(3)$ | $\neq 2, 3$ | | $+$ |
| 40 | $4_1.L_3(4)$ | 3 | $i1$ | $\circ$ |
| 40 | $U_4(2)$ | 0, 5 | $z3$ | $\circ$ |
| 40 | $S_4(5)$ | $\neq 5$ | | $+$ |
| 40 | $S_4(9)$ | 2 | | $-$ |
| 40 | $2.S_4(9)$ | $\neq 2, 3$ | | $-$ |
| 40 | $2.S_6(2)$ | 7 | | $+$ |
| 40 | $S_8(3)$ | 2 | $z3$ | $\circ$ |
| 40 | $2.S_8(3)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 40 | 2.Sz(8) | 7 | | $+$ |
| 40 | 2.Sz(8) | $\neq 2, 7$ | $y7$ | $+$ |
| 41 | $\mathfrak{A}_9$ | 3 | | $+$ |
| 41 | $\mathfrak{A}_{10}$ | 3 | | $+$ |
| 41 | $S_4(9)$ | $\neq 2, 3$ | | $+$ |
| 41 | $S_8(3)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 41 | $J_2$ | 5 | | $+$ |
| 42 | $\mathfrak{A}_9$ | 0, 7 | | $+$ |
| 42 | $\mathfrak{A}_{10}$ | 0, 7 | | $+$ |
| 42 | $6.L_3(4)$ | 0, 7 | $z3, b5$ | $\circ$ |
| 42 | $U_3(7)$ | $\neq 7$ | | $-$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

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| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 42 | $U_7(2)$ | $\neq 2$ | | $-$ |
| 43 | $\mathfrak{A}_8$ | 5 | | $+$ |
| 43 | $\mathfrak{A}_{11}$ | 5 | | $+$ |
| 43 | $\mathfrak{A}_{12}$ | 5 | | $+$ |
| 43 | $U_3(7)$ | $\neq 2, 7$ | | $+$ |
| 43 | $U_3(7)$ | $\neq 2, 7$ | $i1$ | $\circ$ |
| 43 | $U_3(7)$ | $\neq 2, 7$ | $z8$ | $\circ$ |
| 43 | $U_5(2)$ | 5 | | $+$ |
| 43 | $U_7(2)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 43 | $J_1$ | 19 | $b5$ | $+$ |
| 44 | $\mathfrak{A}_{11}$ | $\neq 3, 5$ | | $+$ |
| 44 | $\mathfrak{A}_{12}$ | 2 | | $+$ |
| 44 | $4_2.L_3(4)$ | 7 | $i1$ | $\circ$ |
| 44 | $U_5(2)$ | $\neq 2, 5$ | | $+$ |
| 44 | $M_{11}$ | $\neq 3, 5$ | | $+$ |
| 44 | $M_{12}$ | 2 | | $+$ |
| 44 | $2.M_{12}$ | $\neq 2, 5$ | $i5$ | $\circ$ |
| 44 | $M_{23}$ | 2 | $b7$ | $\circ$ |
| 44 | $M_{24}$ | 2 | $b7$ | $\circ$ |
| 45 | $\mathfrak{A}_8$ | 7 | | $+$ |
| 45 | $\mathfrak{A}_8$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 45 | $\mathfrak{A}_{11}$ | $\neq 2, 11$ | | $+$ |
| 45 | $\mathfrak{A}_{12}$ | 3 | | $+$ |
| 45 | $L_3(4)$ | 7 | | $+$ |
| 45 | $L_3(4)$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 45 | $3.L_3(4)$ | $0, 5$ | $z3, b7$ | $\circ$ |
| 45 | $U_4(2)$ | $0, 5$ | $z3$ | $\circ$ |
| 45 | $3_2.U_4(3)$ | 7 | $z3$ | $\circ$ |
| 45 | $3_2.U_4(3)$ | $\neq 3, 7$ | $z3, b7$ | $\circ$ |
| 45 | $M_{11}$ | $\neq 2, 11$ | | $+$ |
| 45 | $M_{12}$ | $\neq 2, 11$ | | $+$ |
| 45 | $M_{22}$ | 7 | | $+$ |
| 45 | $M_{22}$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 45 | $3.M_{22}$ | 7 | $z3$ | $\circ$ |
| 45 | $3.M_{22}$ | $\neq 3, 7$ | $z3, b7$ | $\circ$ |
| 45 | $M_{23}$ | 7 | | $+$ |
| 45 | $M_{23}$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 45 | $M_{24}$ | 7 | | $+$ |
| 45 | $M_{24}$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 45 | $J_1$ | 7 | $c19$ | $+$ |
| 45 | $3.McL$ | 5 | $z3, b7$ | $\circ$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

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| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 45 | $3.\mathrm{O'N}$ | 7 | | $\circ$ |
| 47 | $\mathfrak{A}_9$ | 7 | | $+$ |
| 48 | $2.\mathfrak{A}_8$ | $\neq 2$ | | $-$ |
| 48 | $\mathfrak{A}_9$ | $0, 2$ | | $+$ |
| 48 | $2.\mathfrak{A}_9$ | 3 | | $+$ |
| 48 | $2.\mathfrak{A}_9$ | $\neq 2, 3$ | $i6$ | $\circ$ |
| 48 | $\mathfrak{A}_{10}$ | 2 | | $+$ |
| 48 | $2.\mathfrak{A}_{10}$ | 3 | | $+$ |
| 48 | $2.\mathfrak{A}_{10}$ | $\neq 2, 3$ | $i6$ | $\circ$ |
| 48 | $12_1.\mathrm{L}_3(4)$ | $0, 7$ | $z12, b5$ | $\circ$ |
| 48 | $12_2.\mathrm{L}_3(4)$ | $0, 7$ | $z12, b5$ | $\circ$ |
| 48 | $12_2.\mathrm{L}_3(4)$ | 5 | $z12$ | $\circ$ |
| 48 | $3.\mathrm{U}_3(5)$ | $\neq 3, 5$ | $z3$ | $\circ$ |
| 48 | $2.\mathrm{S}_6(2)$ | $\neq 2, 7$ | | $+$ |
| 48 | $\mathrm{O}_8^+(2)$ | 3 | | $+$ |
| 48 | $2.\mathrm{Sz}(8)$ | 5 | $c_{13}$ | $+$ |
| 48 | $12.\mathrm{M}_{22}$ | 5 | $z12, b11$ | $\circ$ |
| 49 | $\mathrm{S}_4(4)$ | 17 | | $+$ |
| 49 | $\mathrm{S}_6(2)$ | 3 | | $+$ |
| 49 | $\mathrm{M}_{22}$ | 3 | $b11$ | $\circ$ |
| 49 | $\mathrm{J}_1$ | 11 | $b5, c19$ | $+$ |
| 49 | HS | 3 | $i5, b11$ | $\circ$ |
| 50 | $\mathrm{S}_4(4)$ | $\neq 2, 17$ | | $+$ |
| 50 | $\mathrm{S}_8(2)$ | 3 | | $+$ |
| 50 | $\mathrm{O}_8^+(2)$ | $\neq 2, 3$ | | $+$ |
| 50 | $\mathrm{O}_8^-(2)$ | 3 | | $+$ |
| 50 | $2.\mathrm{J}_2$ | 3 | $b5$ | $-$ |
| 50 | $2.\mathrm{J}_2$ | $\neq 2, 3$ | $i1$ | $\circ$ |
| 50 | He | 7 | | $+$ |
| 51 | $\mathrm{U}_4(4)$ | 5 | | $+$ |
| 51 | $\mathrm{U}_4(4)$ | $\neq 2, 5$ | $z5$ | $\circ$ |
| 51 | $\mathrm{S}_4(4)$ | $\neq 2, 5$ | $b5$ | $+$ |
| 51 | $\mathrm{S}_8(2)$ | $\neq 2, 3$ | | $+$ |
| 51 | $\mathrm{O}_8^-(2)$ | $\neq 2, 3$ | | $+$ |
| 51 | He | $\neq 7$ | $b7$ | $\circ$ |
| 52 | $\mathrm{L}_4(3)$ | $\neq 2, 3$ | | $+$ |
| 52 | $\mathrm{U}_3(4)$ | $\neq 2, 5$ | $z5$ | $\circ$ |
| 52 | $\mathrm{U}_4(4)$ | $\neq 2, 5$ | | $+$ |
| 52 | $2.\mathrm{S}_4(5)$ | $\neq 2, 5$ | $b5$ | $-$ |
| 52 | ${}^3D_4(2)$ | $\neq 2$ | | $+$ |
| 52 | $2.F_4(2)$ | $\neq 2$ | | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

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| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 53 | $\mathfrak{A}_{12}$ | 11 | | $+$ |
| 53 | $\mathfrak{A}_{13}$ | 11 | | $+$ |
| 53 | $M_{12}$ | 11 | | $+$ |
| 54 | $\mathfrak{A}_{12}$ | 0, 3, 7 | | $+$ |
| 54 | $6.L_3(4)$ | 7 | $z3$ | $\circ$ |
| 54 | $M_{12}$ | 0, 3 | | $+$ |
| 54 | $M_{22}$ | 7 | | $+$ |
| 54 | $6.M_{22}$ | 7 | $z3$ | $\circ$ |
| 55 | $\mathfrak{A}_{10}$ | 5 | | $+$ |
| 55 | $\mathfrak{A}_{11}$ | 5 | | $+$ |
| 55 | $\mathfrak{A}_{12}$ | $\neq 2, 3$ | | $+$ |
| 55 | $\mathfrak{A}_{13}$ | 13 | | $+$ |
| 55 | $L_3(7)$ | 3, 19 | | $+$ |
| 55 | $U_5(2)$ | $\neq 2$ | | $+$ |
| 55 | $U_5(2)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 55 | $M_{11}$ | $\neq 2, 3$ | | $+$ |
| 55 | $M_{12}$ | $\neq 2, 3$ | | $+$ |
| 55 | $M_{22}$ | $\neq 2, 7$ | | $+$ |
| 55 | $J_1$ | 19 | $b5$ | $+$ |
| 55 | HS | 5 | | $+$ |
| 56 | $\mathfrak{A}_8$ | 0, 7 | | $+$ |
| 56 | $2.\mathfrak{A}_8$ | 0, 7 | $z3$ | $\circ$ |
| 56 | $2.\mathfrak{A}_8$ | 0, 7 | $i15$ | $\circ$ |
| 56 | $\mathfrak{A}_9$ | $\neq 2, 3$ | | $+$ |
| 56 | $2.\mathfrak{A}_9$ | $\neq 2, 3$ | | $+$ |
| 56 | $\mathfrak{A}_{10}$ | 5 | | $+$ |
| 56 | $2.\mathfrak{A}_{10}$ | 5 | $r6, r21$ | $+$ |
| 56 | $2.\mathfrak{A}_{11}$ | 5 | $r6, r21$ | $+$ |
| 56 | $4_1.L_3(4)$ | $\neq 2, 3$ | $i1$ | $\circ$ |
| 56 | $L_3(7)$ | 0, 2 | | $+$ |
| 56 | $U_3(8)$ | $\neq 2$ | | $-$ |
| 56 | $2.U_4(3)$ | $\neq 2, 3$ | | $+$ |
| 56 | $2.U_6(2)$ | $\neq 2$ | | $+$ |
| 56 | $S_6(2)$ | $\neq 2, 3$ | | $+$ |
| 56 | $2.S_6(2)$ | 3 | $i5$ | $\circ$ |
| 56 | $2.O_8^+(2)$ | $\neq 2$ | | $+$ |
| 56 | $2.Sz(8)$ | $\neq 2, 13$ | $c_{13}$ | $+$ |
| 56 | $2.M_{22}$ | $\neq 2, 5$ | | $+$ |
| 56 | $4.M_{22}$ | $\neq 2$ | $z8$ | $\circ$ |
| 56 | $J_1$ | 2 | $b5$ | $-$ |
| 56 | $J_1$ | 5 | | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

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| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 56 | $J_1$ | $\neq 5, 19$ | $b5$ | $+$ |
| 56 | $2.J_2$ | 5 | | $-$ |
| 56 | $2.J_2$ | 0, 7 | $b5$ | $-$ |
| 56 | HS | 2 | | $+$ |
| 56 | 2.HS | $\neq 2, 5$ | | $+$ |
| 57 | $L_3(7)$ | $\neq 2, 7$ | | $+$ |
| 57 | $3.L_3(7)$ | $\neq 3, 7$ | $z3$ | $\circ$ |
| 57 | $U_3(8)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 57 | $3.U_3(8)$ | $\neq 2, 3$ | $z9$ | $\circ$ |
| 57 | $J_2$ | 3 | $b5$ | $+$ |
| 58 | $U_4(2)$ | 5 | | $+$ |
| 58 | $2.J_2$ | 7 | $b5$ | $-$ |
| 60 | $4_2.L_3(4)$ | 3 | $i1, r7$ | $\circ$ |
| 60 | $6.L_3(4)$ | 0, 5 | $z3, b7$ | $\circ$ |
| 60 | $12_2.L_3(4)$ | 0, 5 | $z12, b7$ | $\circ$ |
| 60 | $U_4(2)$ | 0, 5 | | $+$ |
| 60 | $2.U_4(2)$ | 0, 5 | | $-$ |
| 60 | $2.U_4(2)$ | 0, 5 | $z3$ | $\circ$ |
| 60 | $U_5(3)$ | $\neq 3$ | | $-$ |
| 60 | $S_4(11)$ | 2 | $b11$ | $\circ$ |
| 60 | $2.S_4(11)$ | $\neq 2, 11$ | $b11$ | $\circ$ |
| 61 | $L_6(2)$ | 3, 7 | | $+$ |
| 61 | $U_5(3)$ | $\neq 2, 3$ | $i1$ | $\circ$ |
| 61 | $S_4(11)$ | $\neq 2, 11$ | $b11$ | $\circ$ |
| 62 | $L_6(2)$ | 0, 5, 31 | | $+$ |
| 62 | $S_6(5)$ | $\neq 5$ | $b5$ | $-$ |
| 63 | $L_3(4)$ | 5 | | $+$ |
| 63 | $L_3(4)$ | $\neq 2, 5$ | $b5$ | $+$ |
| 63 | $3.L_3(4)$ | 5 | $z3$ | $\circ$ |
| 63 | $3.L_3(4)$ | 0, 7 | $z3, b5$ | $\circ$ |
| 63 | $U_3(4)$ | 13 | | $+$ |
| 63 | $2.S_6(5)$ | $\neq 2, 5$ | $b5$ | $+$ |
| 63 | $Sz(8)$ | 5 | | $+$ |
| 63 | $J_2$ | $\neq 2, 5$ | | $+$ |
| 64 | $\mathfrak{A}_8$ | 0 | | $+$ |
| 64 | $2.\mathfrak{A}_8$ | 0 | | $-$ |
| 64 | $\mathfrak{A}_{10}$ | 2 | | $+$ |
| 64 | $2.\mathfrak{A}_{10}$ | 0, 7 | | $+$ |
| 64 | $\mathfrak{A}_{13}$ | 2 | | $-$ |
| 64 | $\mathfrak{A}_{13}$ | 3 | | $+$ |
| 64 | $\mathfrak{A}_{14}$ | 2, 3 | | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

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| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 64 | $2.\mathfrak{A}_{14}$ | $\neq 2, 7$ | | $-$ |
| 64 | $\mathfrak{A}_{15}$ | 2 | $b15$ | $\circ$ |
| 64 | $2.\mathfrak{A}_{15}$ | 3, 5 | | $-$ |
| 64 | $2.\mathfrak{A}_{15}$ | $\neq 2, 3, 5$ | $b15$ | $\circ$ |
| 64 | $\mathfrak{A}_{16}$ | 2 | $b7, b15, b39, b55$ | $\circ$ |
| 64 | $L_3(4)$ | 0 | | $+$ |
| 64 | $2.L_3(4)$ | 0, 7 | | $+$ |
| 64 | $4_1.L_3(4)$ | 0, 7 | $i1$ | $\circ$ |
| 64 | $4_2.L_3(4)$ | 0 | $i1$ | $\circ$ |
| 64 | $U_3(4)$ | 0, 3 | | $+$ |
| 64 | $U_4(2)$ | 0 | | $+$ |
| 64 | $2.U_4(2)$ | 0 | | $-$ |
| 64 | $S_4(5)$ | 2 | | $-$ |
| 64 | $S_4(5)$ | 3 | | $+$ |
| 64 | $2.S_6(2)$ | 5 | | $+$ |
| 64 | $2.S_6(2)$ | 0, 7 | $i5$ | $\circ$ |
| 64 | $Sz(8)$ | 0, 7 | | $+$ |
| 64 | $2.Sz(8)$ | 0, 7 | | $+$ |
| 64 | $G_2(3)$ | $\neq 3$ | $z3$ | $\circ$ |
| 64 | $G_2(4)$ | 3 | | $+$ |
| 64 | $2.M_{22}$ | 11 | | $+$ |
| 64 | $4.M_{22}$ | 3 | $i1$ | $\circ$ |
| 64 | $J_1$ | 11 | $b5, c19$ | $+$ |
| 64 | $J_2$ | 2 | $b5$ | $+$ |
| 64 | $2.J_2$ | 5 | | $-$ |
| 64 | $2.J_2$ | 0 | $b5$ | $-$ |
| 64 | Suz | 3 | | $+$ |
| 65 | $\mathfrak{A}_{13}$ | $\neq 2, 3, 11$ | | $+$ |
| 65 | $L_4(3)$ | $\neq 2, 3$ | | $+$ |
| 65 | $U_3(4)$ | 0, 13 | $z5$ | $\circ$ |
| 65 | $U_3(4)$ | $\neq 2, 3$ | | $+$ |
| 65 | $S_4(5)$ | 0, 13 | | $+$ |
| 65 | $Sz(8)$ | $\neq 2, 7$ | $y7$ | $+$ |
| 65 | $G_2(4)$ | $\neq 2, 3$ | | $+$ |
| 66 | $\mathfrak{A}_{10}$ | 7 | | $+$ |
| 66 | $\mathfrak{A}_{11}$ | 7 | | $+$ |
| 66 | $\mathfrak{A}_{13}$ | $\neq 2, 13$ | | $+$ |
| 66 | $\mathfrak{A}_{14}$ | 7 | | $+$ |
| 66 | $U_5(2)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 66 | $M_{12}$ | $\neq 2, 3$ | | $+$ |
| 66 | $6.M_{22}$ | 7 | $z3$ | $\circ$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

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| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 66 | $6.M_{22}$ | 0, 5, 11 | $z3, b7$ | ∘ |
| 66 | 3.Suz | $\neq 3$ | $z3$ | ∘ |
| 69 | $J_1$ | 11 | $b5, c19$ | + |
| 70 | $\mathfrak{A}_8$ | $\neq 2, 3$ | | + |
| 70 | $2.L_3(4)$ | $\neq 2, 3$ | | + |
| 70 | $U_4(3)$ | 2 | $z3$ | ∘ |
| 70 | $2.U_4(3)$ | $\neq 2, 3$ | | + |
| 70 | $2.U_4(3)$ | $\neq 2, 3$ | $z3$ | ∘ |
| 70 | $S_6(2)$ | $\neq 2, 3$ | | + |
| 70 | $M_{22}$ | 2 | $b11$ | ∘ |
| 70 | $J_2$ | 5 | | + |
| 70 | $J_2$ | 0, 7 | $b5$ | + |
| 71 | $L_3(8)$ | 73 | | + |
| 72 | $2.\mathfrak{A}_9$ | 7 | $z3, r2$ | ∘ |
| 72 | $L_3(8)$ | $\neq 2, 73$ | | + |
| 72 | $U_3(9)$ | $\neq 3$ | | − |
| 73 | $L_3(8)$ | $\neq 2, 7$ | $z7$ | ∘ |
| 73 | $U_3(9)$ | $\neq 2, 3$ | | + |
| 73 | $U_3(9)$ | $\neq 3, 5$ | $z5$ | ∘ |
| 75 | $\mathfrak{A}_{10}$ | 0, 5 | | + |
| 75 | $U_3(4)$ | $\neq 2, 13$ | $d13$ | ∘ |
| 75 | $J_1$ | 7 | | + |
| 76 | $\mathfrak{A}_{14}$ | 13 | | + |
| 76 | $\mathfrak{A}_{15}$ | 13 | | + |
| 76 | $J_1$ | 2 | | − |
| 76 | $J_1$ | $\neq 7, 11$ | | + |
| 77 | $\mathfrak{A}_{14}$ | $\neq 2, 3, 13$ | | + |
| 77 | ${}^2F_4(2)'$ | 3 | | + |
| 77 | $J_1$ | $\neq 2, 3, 19$ | | + |
| 77 | $J_1$ | $\neq 2, 5$ | $b5$ | + |
| 77 | HS | $\neq 2, 5$ | | + |
| 77 | $Fi_{22}$ | 3 | | + |
| 78 | $\mathfrak{A}_9$ | 2 | | + |
| 78 | $\mathfrak{A}_{14}$ | $\neq 2, 7$ | | + |
| 78 | $\mathfrak{A}_{15}$ | 3, 5 | | + |
| 78 | $6_1.U_4(3)$ | 5 | $z3$ | ∘ |
| 78 | $S_4(5)$ | $\neq 2, 5$ | $b5$ | + |
| 78 | $S_6(3)$ | $\neq 3$ | | + |
| 78 | $O_7(3)$ | $\neq 3$ | | + |
| 78 | $G_2(3)$ | $\neq 3$ | | + |
| 78 | $G_2(4)$ | $\neq 2$ | | + |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 78 | $^2F_4(2)'$ | $\neq 2,3$ | | $+$ |
| 78 | $\mathrm{M}_{12}$ | 5 | | $+$ |
| 78 | $3.\mathrm{M}_{22}$ | 5 | $z3$ | $\circ$ |
| 78 | $\mathrm{J}_3$ | 2 | $b5, b17$ | $+$ |
| 78 | Suz | 3 | | $+$ |
| 78 | 3.Suz | $\neq 2,3$ | $z3$ | $\circ$ |
| 78 | $\mathrm{Fi}_{22}$ | 2 | | $+$ |
| 78 | $\mathrm{Fi}_{22}$ | $\neq 2,3$ | | $+$ |
| 80 | $4_1.\mathrm{L}_3(4)$ | $0,5$ | $i1, r7$ | $\circ$ |
| 80 | $4_2.\mathrm{L}_3(4)$ | $0,5$ | $i1, r7$ | $\circ$ |
| 80 | $2.\mathrm{U}_4(2)$ | $0,5$ | | $-$ |
| 80 | $\mathrm{J}_3$ | 2 | $b17$ | $+$ |
| 81 | $\mathrm{U}_4(2)$ | 0 | | $+$ |
| 83 | $\mathfrak{A}_9$ | 5 | | $+$ |
| 83 | $\mathrm{L}_4(4)$ | $5,17$ | | $+$ |
| 83 | $\mathrm{S}_6(2)$ | 5 | | $+$ |
| 83 | $\mathrm{O}_8^+(2)$ | 5 | | $+$ |
| 83 | $\mathrm{O}_8^-(2)$ | 17 | | $+$ |
| 84 | $\mathfrak{A}_9$ | $0,7$ | | $+$ |
| 84 | $\mathfrak{A}_{10}$ | $\neq 2,5$ | | $+$ |
| 84 | $\mathfrak{A}_{11}$ | 11 | | $+$ |
| 84 | $3.\mathrm{L}_3(4)$ | $0,7$ | $z3$ | $\circ$ |
| 84 | $12_2.\mathrm{L}_3(4)$ | $0,7$ | $z12$ | $\circ$ |
| 84 | $\mathrm{L}_4(4)$ | $0,3,7$ | | $+$ |
| 84 | $\mathrm{U}_3(5)$ | $\neq 2,5$ | | $+$ |
| 84 | $3.\mathrm{U}_3(5)$ | $\neq 3,5$ | $z3$ | $\circ$ |
| 84 | $3_1.\mathrm{U}_4(3)$ | 2 | $z3$ | $\circ$ |
| 84 | $6_1.\mathrm{U}_4(3)$ | $0,7$ | $z3$ | $\circ$ |
| 84 | $12_1.\mathrm{U}_4(3)$ | $\neq 2,3$ | $z12$ | $\circ$ |
| 84 | $\mathrm{S}_4(13)$ | 2 | $b13$ | $-$ |
| 84 | $2.\mathrm{S}_4(13)$ | $\neq 2,13$ | $b13$ | $-$ |
| 84 | $\mathrm{S}_6(2)$ | $0,7$ | | $+$ |
| 84 | $\mathrm{O}_8^+(2)$ | $0,7$ | | $+$ |
| 84 | $\mathrm{O}_8^-(2)$ | $0,5,7$ | | $+$ |
| 84 | $2.\mathrm{M}_{12}$ | 3 | $i5, b11$ | $\circ$ |
| 84 | $3.\mathrm{M}_{22}$ | 11 | $z3$ | $\circ$ |
| 84 | $3.\mathrm{M}_{22}$ | 2 | $z3, b11$ | $\circ$ |
| 84 | $\mathrm{J}_2$ | 2 | | $+$ |
| 84 | $2.\mathrm{J}_2$ | $0,7$ | | $-$ |
| 84 | $\mathrm{J}_3$ | $2,3$ | $b19$ | $\circ$ |
| 85 | $\mathrm{L}_4(4)$ | $\neq 2,3$ | $z3$ | $\circ$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 85 | $U_8(2)$ | 3 | | $+$ |
| 85 | $U_8(2)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 85 | $S_4(4)$ | $\neq 2, 3$ | | $+$ |
| 85 | $S_4(13)$ | $\neq 2, 13$ | $b13$ | $+$ |
| 85 | $S_8(2)$ | $\neq 2, 3$ | | $+$ |
| 85 | $J_2$ | 5 | | $+$ |
| 85 | $J_3$ | 19 | | $+$ |
| 85 | $J_3$ | 0, 5, 17 | $b19$ | $\circ$ |
| 86 | $U_8(2)$ | $\neq 2, 3$ | | $+$ |
| 88 | $4.M_{22}$ | 5 | $z8, b7$ | $\circ$ |
| 89 | $\mathfrak{A}_{10}$ | 7 | | $+$ |
| 89 | $\mathfrak{A}_{11}$ | 5 | | $+$ |
| 89 | $\mathfrak{A}_{12}$ | 5 | | $+$ |
| 89 | $\mathfrak{A}_{15}$ | 7 | | $+$ |
| 89 | $\mathfrak{A}_{16}$ | 7 | | $+$ |
| 89 | $L_3(9)$ | 7, 13 | | $+$ |
| 89 | $L_4(3)$ | 13 | | $+$ |
| 89 | $U_4(3)$ | 7 | | $+$ |
| 89 | $S_4(5)$ | 13 | | $+$ |
| 89 | $J_1$ | 7 | $c19$ | $+$ |
| 89 | $J_2$ | 7 | | $+$ |
| 90 | $\mathfrak{A}_{10}$ | 0, 3 | | $+$ |
| 90 | $\mathfrak{A}_{15}$ | $\neq 7, 13$ | | $+$ |
| 90 | $\mathfrak{A}_{16}$ | 2 | | $+$ |
| 90 | $2.L_3(4)$ | $\neq 2, 7$ | | $+$ |
| 90 | $6.L_3(4)$ | 0, 5 | $z3$ | $\circ$ |
| 90 | $L_3(9)$ | 0, 2, 5 | | $+$ |
| 90 | $L_4(3)$ | 0, 5 | | $+$ |
| 90 | $U_4(3)$ | 0, 5 | | $+$ |
| 90 | $3_1.U_4(3)$ | 2 | $z3$ | $\circ$ |
| 90 | $6_2.U_4(3)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 90 | $S_4(5)$ | 0, 3 | | $+$ |
| 90 | $O_7(3)$ | 2 | | $+$ |
| 90 | $G_2(3)$ | 2 | | $+$ |
| 90 | $6.M_{22}$ | 11 | $z12$ | $\circ$ |
| 90 | $J_2$ | $\neq 2, 7$ | | $+$ |
| 91 | $\mathfrak{A}_{15}$ | $\neq 2, 3, 5$ | | $+$ |
| 91 | $L_3(9)$ | $\neq 2, 3$ | | $+$ |
| 91 | $L_3(9)$ | $\neq 2, 3$ | $i1$ | $\circ$ |
| 91 | $L_3(9)$ | $\neq 2, 3$ | $z8$ | $\circ$ |
| 91 | $S_6(2)$ | 3 | | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 91 | $S_6(3)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 91 | $O_7(3)$ | $\neq 2, 3$ | | $+$ |
| 91 | Sz(8) | $\neq 2, 5$ | | $+$ |
| 91 | $G_2(3)$ | $\neq 2, 3$ | | $+$ |
| 91 | $M_{12}$ | 11 | | $+$ |
| 92 | $2.G_2(4)$ | 5 | | $-$ |
| 94 | $L_5(2)$ | 7 | | $+$ |
| 94 | $S_6(2)$ | 7 | | $+$ |
| 96 | $L_3(5)$ | 31 | | $+$ |
| 96 | $L_3(5)$ | $\neq 5, 31$ | $x31$ | $\circ$ |
| 96 | $L_3(7)$ | 3 | | $+$ |
| 96 | $3.L_3(7)$ | $\neq 3, 7$ | $z3$ | $\circ$ |
| 96 | $12.M_{22}$ | 11 | $z12$ | $\circ$ |
| 98 | $S_6(2)$ | 3 | | $+$ |
| 98 | $M_{12}$ | 5 | | $+$ |
| 98 | $M_{22}$ | 2 | | $-$ |
| 98 | $M_{22}$ | 5 | | $+$ |
| 98 | HS | 5 | | $+$ |
| 99 | $M_{12}$ | $\neq 2, 5$ | | $+$ |
| 99 | $M_{22}$ | 0, 3, 11 | | $+$ |
| 99 | $3.M_{22}$ | 0, 7, 11 | $z3$ | $\circ$ |
| 100 | $\mathfrak{A}_{11}$ | 2 | | $+$ |
| 100 | $\mathfrak{A}_{12}$ | 2 | | $+$ |
| 100 | $U_5(2)$ | 3 | | $+$ |
| 101 | $\mathfrak{A}_9$ | 7 | | $+$ |
| 101 | $\mathfrak{A}_{10}$ | 7 | | $+$ |
| 101 | $J_2$ | 7 | | $+$ |
| 101 | He | 2 | $b7$ | $\circ$ |
| 103 | $\mathfrak{A}_{16}$ | 3, 5 | | $+$ |
| 103 | $\mathfrak{A}_{17}$ | 3, 5 | | $+$ |
| 103 | $G_2(3)$ | 7 | | $+$ |
| 104 | $2.\mathfrak{A}_9$ | 3 | | $+$ |
| 104 | $\mathfrak{A}_{16}$ | 0, 11, 13 | | $+$ |
| 104 | $U_3(5)$ | 2 | | $-$ |
| 104 | $U_4(5)$ | 2 | | $-$ |
| 104 | $U_4(5)$ | 3 | | $+$ |
| 104 | $U_4(5)$ | $\neq 3, 5$ | $z3$ | $\circ$ |
| 104 | $2.U_4(5)$ | 0, 7, 13 | $z3$ | $\circ$ |
| 104 | $2.U_4(5)$ | $\neq 2, 5$ | | $-$ |
| 104 | $S_4(5)$ | $\neq 5$ | | $+$ |
| 104 | $2.S_4(5)$ | 0, 13 | | $-$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 104 | $2.S_6(2)$ | 3, 5 | | + |
| 104 | $O_7(3)$ | 2 | | + |
| 104 | $2.O_8^+(2)$ | 3, 5 | | + |
| 104 | 2.Sz(8) | 0, 13 | | + |
| 104 | $G_2(3)$ | 0, 13 | | + |
| 104 | $2.G_2(4)$ | $\neq 2, 5$ | $b5$ | − |
| 104 | $M_{23}$ | 3 | $b11$, $b23$ | ∘ |
| 104 | McL | 3 | $b11$ | ∘ |
| 104 | He | 5 | $r21$ | + |
| 105 | $\mathfrak{A}_9$ | $\neq 2, 3$ | | + |
| 105 | $\mathfrak{A}_{16}$ | $\neq 2$ | | + |
| 105 | $\mathfrak{A}_{17}$ | 17 | | + |
| 105 | $U_3(5)$ | 0, 7 | | + |
| 105 | $3.U_3(5)$ | 0, 7 | $z3$ | ∘ |
| 105 | $3_1.U_4(3)$ | $\neq 2, 3$ | $z3$ | ∘ |
| 105 | $U_4(5)$ | 0, 7, 13 | | + |
| 105 | $S_6(2)$ | $\neq 2, 3$ | | + |
| 105 | $S_6(3)$ | $\neq 2, 3$ | | + |
| 105 | $O_7(3)$ | $\neq 2, 3$ | | + |
| 105 | $3.M_{22}$ | 0, 5, 7 | $z3$, $b11$ | ∘ |
| 106 | $J_1$ | 11 | $b5$, $c19$ | + |
| 108 | $2.M_{12}$ | 11 | | + |
| 109 | $\mathfrak{A}_{11}$ | 3 | | + |
| 109 | ${}^2F_4(2)'$ | 5 | $r2$, $r3$, $b13$ | + |
| 110 | $\mathfrak{A}_{11}$ | 0, 5, 11 | | + |
| 110 | $U_3(11)$ | $\neq 11$ | | − |
| 110 | $U_5(2)$ | $\neq 2$ | | − |
| 110 | $U_5(2)$ | $\neq 2, 3$ | $z3$ | ∘ |
| 110 | $2.M_{12}$ | $\neq 2, 3$ | $i2$ | ∘ |
| 110 | $J_3$ | 19 | $b17$, $y9$ | + |
| 110 | Suz | 2 | $b5$, $b13$, $r21$ | + |
| 111 | $U_3(11)$ | $\neq 2, 11$ | | + |
| 111 | $U_3(11)$ | $\neq 2, 11$ | $i1$ | ∘ |
| 111 | $3.U_3(11)$ | $\neq 3, 11$ | $z3$ | ∘ |
| 111 | $3.U_3(11)$ | 0, 5, 37 | $z12$ | ∘ |
| 111 | Ly | 5 | | + |
| 112 | $2.\mathfrak{A}_9$ | 0, 7 | | + |
| 112 | $2.S_6(2)$ | 0, 7 | | + |
| 112 | $2.O_8^+(2)$ | 0, 7 | | + |
| 112 | $J_4$ | 2 | | + |
| 114 | $6_1.U_4(3)$ | 7 | $z3$ | ∘ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

Continued from the previous page

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 115 | $\mathfrak{A}_9$ | 7 | | + |
| 118 | $\mathfrak{A}_{17}$ | 2 | | + |
| 118 | $\mathfrak{A}_{18}$ | 2 | | + |
| 118 | $S_8(2)$ | 3, 5 | | + |
| 119 | $\mathfrak{A}_{17}$ | $\neq 2, 3, 5$ | | + |
| 119 | $L_5(3)$ | 11 | | + |
| 119 | $U_5(2)$ | 11 | | + |
| 119 | $S_8(2)$ | 0, 7, 17 | | + |
| 119 | $J_1$ | 11 | $c19$ | + |
| 120 | $\mathfrak{A}_9$ | 0, 5 | | + |
| 120 | $2.\mathfrak{A}_9$ | 0, 5 | $z3$ | ∘ |
| 120 | $\mathfrak{A}_{11}$ | $\neq 2, 11$ | | + |
| 120 | $\mathfrak{A}_{12}$ | 3 | | + |
| 120 | $\mathfrak{A}_{17}$ | $\neq 2, 17$ | | + |
| 120 | $\mathfrak{A}_{18}$ | 3 | | + |
| 120 | $12_1.L_3(4)$ | 0, 5 | $z12$ | ∘ |
| 120 | $L_5(3)$ | $\neq 3, 11$ | | + |
| 120 | $U_4(3)$ | 2 | | + |
| 120 | $2.U_4(3)$ | $\neq 2, 3$ | | + |
| 120 | $4.U_4(3)$ | $\neq 2, 3$ | $i1$ | ∘ |
| 120 | $6_1.U_4(3)$ | 0, 5 | $z3$ | ∘ |
| 120 | $12_1.U_4(3)$ | $\neq 2, 3$ | $z12$ | ∘ |
| 120 | $U_5(2)$ | 0, 5 | | + |
| 120 | $2.U_6(2)$ | 3 | | + |
| 120 | $6.U_6(2)$ | $\neq 2, 3$ | $z3$ | ∘ |
| 120 | $S_6(2)$ | 0, 5 | | + |
| 120 | $2.S_6(2)$ | $\neq 2, 3$ | | + |
| 120 | $M_{12}$ | 0, 5 | | + |
| 120 | $2.M_{12}$ | 0, 5 | | + |
| 120 | $2.M_{22}$ | $\neq 2, 11$ | | + |
| 120 | $6.M_{22}$ | 0, 5, 11 | $z3$ | ∘ |
| 120 | $12.M_{22}$ | $\neq 2, 3$ | $z24$ | ∘ |
| 120 | $M_{23}$ | 2 | | + |
| 120 | $M_{24}$ | 2 | | + |
| 120 | $J_1$ | $\neq 11, 19$ | $c19$ | + |
| 120 | $2.HS$ | 5 | $i1$ | ∘ |
| 121 | $L_5(3)$ | $\neq 2, 3$ | | + |
| 121 | $S_{10}(3)$ | $\neq 3$ | $z3$ | ∘ |
| 122 | $2.S_{10}(3)$ | $\neq 2, 3$ | $z3$ | ∘ |
| 123 | $L_5(2)$ | 5 | | + |
| 124 | $\mathfrak{A}_{10}$ | 7 | | + |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 124 | $L_3(5)$ | $\neq 5$ | | $+$ |
| 124 | $L_3(5)$ | 2 | | $-$ |
| 124 | $L_3(5)$ | $\neq 2, 5$ | $i1$ | $\circ$ |
| 124 | $L_3(5)$ | 0, 31 | $y24'$ | $\circ$ |
| 124 | $L_5(2)$ | 0, 3, 31 | | $+$ |
| 124 | $U_3(5)$ | 7 | | $+$ |
| 124 | Sz(32) | $\neq 2$ | $i1$ | $\circ$ |
| 124 | $G_2(5)$ | $\neq 5$ | | $+$ |
| 124 | ${}^2F_4(2)'$ | 3 | $b13$ | $+$ |
| 124 | $J_2$ | 7 | | $+$ |
| 125 | $L_3(5)$ | 0 | | $+$ |
| 125 | $L_7(2)$ | 127 | | $+$ |
| 125 | $U_3(5)$ | 0 | | $+$ |
| 126 | $\mathfrak{A}_{10}$ | $\neq 2, 5$ | | $+$ |
| 126 | $\mathfrak{A}_{11}$ | 11 | | $+$ |
| 126 | $\mathfrak{A}_{11}$ | $\neq 2, 11$ | $b11$ | $\circ$ |
| 126 | $\mathfrak{A}_{12}$ | 3 | $b11, i35$ | $\circ$ |
| 126 | $L_7(2)$ | $\neq 2, 127$ | | $+$ |
| 126 | $U_3(5)$ | $\neq 2, 5$ | | $+$ |
| 126 | $U_3(5)$ | $\neq 2, 5$ | $i2$ | $\circ$ |
| 126 | $3.U_3(5)$ | 0, 7 | $z3$ | $\circ$ |
| 126 | $3.U_3(5)$ | 0, 7 | $z3, i2$ | $\circ$ |
| 126 | $3_2.U_4(3)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 126 | $6_1.U_4(3)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 126 | $6_2.U_4(3)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 126 | $6_2.U_4(3)$ | $\neq 2, 3$ | $z12$ | $\circ$ |
| 126 | $S_4(7)$ | 2 | | $+$ |
| 126 | $S_4(7)$ | $\neq 2, 7$ | | $+$ |
| 126 | $2.M_{22}$ | 11 | | $+$ |
| 126 | $2.M_{22}$ | $\neq 2, 11$ | $b11$ | $\circ$ |
| 126 | $6.M_{22}$ | 0, 5, 7 | $z3, b11$ | $\circ$ |
| 126 | $J_2$ | 0, 7 | | $+$ |
| 126 | $2.J_2$ | $\neq 2, 5$ | $b5$ | $-$ |
| 126 | $3.J_3$ | 2 | $z3, b17, b19$ | $\circ$ |
| 126 | 3.McL | 11 | $z3$ | $\circ$ |
| 126 | 3.McL | $\neq 3, 11$ | $z3, b11$ | $\circ$ |
| 126 | $Co_3$ | 3 | $i5, b11, b23$ | $\circ$ |
| 128 | $2.\mathfrak{A}_{11}$ | 11 | | $+$ |
| 128 | $2.\mathfrak{A}_{12}$ | 11 | | $+$ |
| 128 | $2.\mathfrak{A}_{16}$ | $\neq 2$ | | $+$ |
| 128 | $\mathfrak{A}_{17}$ | 2 | $b17$ | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 128 | $2.\mathfrak{A}_{17}$ | 17 | | $+$ |
| 128 | $2.\mathfrak{A}_{17}$ | $\neq 2, 17$ | $b17$ | $+$ |
| 128 | $2.\mathfrak{A}_{18}$ | 3 | $r2, r5, r14, b17, b65, b77$ | $+$ |
| 130 | $S_4(5)$ | 0, 13 | | $+$ |
| 131 | $\mathfrak{A}_{11}$ | 3, 7 | | $+$ |
| 131 | $\mathfrak{A}_{12}$ | 3, 7 | | $+$ |
| 131 | $L_3(11)$ | 7, 19 | | $+$ |
| 132 | $\mathfrak{A}_{11}$ | 0, 11 | | $+$ |
| 132 | $\mathfrak{A}_{12}$ | 0, 11 | | $+$ |
| 132 | $L_3(11)$ | 0, 2, 3, 5 | | $+$ |
| 132 | $12_1.U_4(3)$ | 5 | $z12$ | $\circ$ |
| 132 | HS | 2 | | $-$ |
| 132 | HN | 2 | $b5$ | $+$ |
| 133 | $\mathfrak{A}_9$ | 5 | | $+$ |
| 133 | $\mathfrak{A}_{10}$ | 5 | $r21$ | $+$ |
| 133 | $\mathfrak{A}_{11}$ | 5 | $r21$ | $+$ |
| 133 | $L_3(11)$ | $\neq 2, 11$ | | $+$ |
| 133 | $L_3(11)$ | $\neq 5, 11$ | $z5$ | $\circ$ |
| 133 | $U_3(8)$ | $\neq 2$ | | $+$ |
| 133 | $S_6(2)$ | 5 | | $+$ |
| 133 | $M_{22}$ | 5 | | $+$ |
| 133 | $J_1$ | $\neq 2, 11$ | | $+$ |
| 133 | $J_1$ | 0, 7, 19 | $b5$ | $+$ |
| 133 | $J_2$ | 3 | | $+$ |
| 133 | HS | 5 | | $+$ |
| 133 | Ru | 5 | | $+$ |
| 133 | HN | 5 | | $+$ |
| 133 | HN | $\neq 2, 5$ | $b5$ | $+$ |
| 134 | $\mathfrak{A}_9$ | 5 | | $+$ |
| 134 | $\mathfrak{A}_{18}$ | 17 | | $+$ |
| 134 | $\mathfrak{A}_{19}$ | 17 | | $+$ |
| 134 | $S_8(2)$ | 17 | | $+$ |
| 135 | $\mathfrak{A}_{18}$ | $\neq 2, 17$ | | $+$ |
| 135 | $S_8(2)$ | $\neq 2, 17$ | | $+$ |
| 136 | $\mathfrak{A}_{18}$ | $\neq 2, 3$ | | $+$ |
| 136 | $\mathfrak{A}_{19}$ | 19 | | $+$ |
| 140 | $U_4(3)$ | $\neq 2, 3$ | | $+$ |
| 140 | $4.U_4(3)$ | $\neq 2, 3$ | $i1$ | $\circ$ |
| 141 | $S_6(2)$ | 5 | | $+$ |
| 142 | Suz | 2 | | $+$ |
| 143 | $\mathfrak{A}_{12}$ | 3 | | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 143 | $\mathfrak{A}_{13}$ | 3 | | $+$ |
| 143 | Suz | $\neq 2, 3$ | | $+$ |
| 144 | $\mathfrak{A}_{11}$ | 2 | | $+$ |
| 144 | $2.\mathfrak{A}_{11}$ | 0, 3, 7 | | $+$ |
| 144 | $\mathfrak{A}_{12}$ | 5 | | $+$ |
| 144 | $\mathfrak{A}_{12}$ | 2 | $i35$, $z3$ | $\circ$ |
| 144 | $2.\mathfrak{A}_{12}$ | 3 | $i2$, $i5$, $r7$ | $\circ$ |
| 144 | $\mathfrak{A}_{13}$ | 5 | | $+$ |
| 144 | $\mathfrak{A}_{13}$ | 2 | $i35$, $z3$ | $\circ$ |
| 144 | $2.\mathfrak{A}_{13}$ | 3 | $i2$, $i5$, $r7$ | $\circ$ |
| 144 | $U_3(5)$ | $\neq 5, 7$ | $b7$ | $\circ$ |
| 144 | $3.U_3(5)$ | 0, 2 | $z3$, $b7$ | $\circ$ |
| 144 | $S_4(17)$ | 2 | $b17$ | $-$ |
| 144 | $2.S_4(17)$ | $\neq 2, 17$ | $b17$ | $-$ |
| 144 | $M_{12}$ | 0, 2 | | $+$ |
| 144 | $4.M_{22}$ | 7 | $i1$ | $\circ$ |
| 144 | $4.M_{22}$ | 0, 3, 11 | $i1$, $r7$ | $\circ$ |
| 144 | $12.M_{22}$ | 7 | $z12$ | $\circ$ |
| 144 | $12.M_{22}$ | 0, 5, 11 | $z12$, $b7$ | $\circ$ |
| 145 | $S_4(17)$ | $\neq 2, 17$ | $b17$ | $+$ |
| 147 | $O_8^+(2)$ | 3 | | $+$ |
| 150 | $3_1.U_4(3)$ | 2 | $z3$, $b7$ | $\circ$ |
| 150 | $S_4(7)$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 151 | $\mathfrak{A}_{19}$ | 3 | | $+$ |
| 151 | $\mathfrak{A}_{20}$ | 3 | | $+$ |
| 152 | $2.\mathfrak{A}_{10}$ | 7 | | $+$ |
| 152 | $\mathfrak{A}_{19}$ | $\neq 3, 17$ | | $+$ |
| 152 | $\mathfrak{A}_{20}$ | 2 | | $+$ |
| 152 | $L_3(7)$ | $\neq 3, 7$ | | $+$ |
| 152 | $2.O_8^+(2)$ | 7 | | $+$ |
| 153 | $\mathfrak{A}_{12}$ | 5 | | $+$ |
| 153 | $\mathfrak{A}_{19}$ | $\neq 2, 19$ | | $+$ |
| 153 | $\mathfrak{A}_{20}$ | 5 | | $+$ |
| 153 | $3_2.U_4(3)$ | 5 | $z3$ | $\circ$ |
| 153 | $S_4(4)$ | $\neq 2$ | | $+$ |
| 153 | $O_{10}^-(2)$ | 3, 5 | | $+$ |
| 153 | $3.M_{22}$ | 5 | $z3$ | $\circ$ |
| 153 | $J_3$ | 3 | $b5$ | $+$ |
| 153 | $3.J_3$ | 5 | $z3$ | $\circ$ |
| 153 | $3.J_3$ | $\neq 3, 5$ | $z3$, $b5$ | $\circ$ |
| 153 | $3.McL$ | 5 | $z3$, $b7$ | $\circ$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 153 | He | 7 | | $+$ |
| 153 | He | $\neq 2, 7$ | $b7$ | $\circ$ |
| 153 | 3.O′N | 2 | $z3$ | $\circ$ |
| 154 | $\mathfrak{A}_{12}$ | 0, 7, 11 | | $+$ |
| 154 | $L_4(5)$ | 2, 3, 13 | | $+$ |
| 154 | $O_8^-(2)$ | 3 | | $+$ |
| 154 | $O_{10}^-(2)$ | $\neq 2, 3, 5$ | | $+$ |
| 154 | $M_{22}$ | 0, 7, 11 | | $+$ |
| 154 | $2.M_{22}$ | $\neq 2, 5$ | $i1$ | $\circ$ |
| 154 | HS | $\neq 2, 5$ | | $+$ |
| 154 | O′N | 3 | $r7$ | $+$ |
| 155 | $\mathfrak{A}_{10}$ | 5 | | $+$ |
| 155 | $\mathfrak{A}_{11}$ | 7 | | $+$ |
| 155 | $L_3(5)$ | 0, 31 | | $+$ |
| 155 | $L_3(5)$ | 0, 31 | $i1$ | $\circ$ |
| 155 | $L_4(5)$ | 0, 31 | | $+$ |
| 155 | $L_5(2)$ | $\neq 2$ | | $+$ |
| 155 | $S_{10}(2)$ | $\neq 2$ | | $+$ |
| 155 | $O_{10}^+(2)$ | $\neq 2$ | | $+$ |
| 156 | $2.L_4(5)$ | $\neq 2, 5$ | | $+$ |
| 156 | $4.L_4(5)$ | $\neq 2, 5$ | $i1$ | $\circ$ |
| 156 | $U_3(13)$ | $\neq 13$ | | $-$ |
| 156 | $S_4(5)$ | $\neq 2, 5$ | | $+$ |
| 156 | $2.S_4(5)$ | $\neq 2, 5$ | | $-$ |
| 157 | $U_3(13)$ | $\neq 2, 13$ | | $+$ |
| 157 | $U_3(13)$ | $\neq 7, 13$ | $z7$ | $\circ$ |
| 160 | $\mathfrak{A}_9$ | 2 | | $+$ |
| 160 | $2.\mathfrak{A}_9$ | 0, 5 | | $+$ |
| 160 | $\mathfrak{A}_{10}$ | $\neq 3, 7$ | | $+$ |
| 160 | $2.\mathfrak{A}_{10}$ | 5 | $r6, r21$ | $+$ |
| 160 | $2.\mathfrak{A}_{12}$ | 0, 5, 7 | $b11$ | $\circ$ |
| 160 | $2.O_8^+(2)$ | 0, 5 | | $+$ |
| 160 | $2.M_{12}$ | 0, 5 | $b11$ | $\circ$ |
| 160 | $4.M_{22}$ | 11 | $i1$ | $\circ$ |
| 160 | $4.M_{22}$ | $\neq 2, 11$ | $i1, r11$ | $\circ$ |
| 160 | $J_2$ | 0, 2 | | $+$ |
| 162 | $\mathfrak{A}_9$ | 0, 3 | | $+$ |
| 162 | $3.G_2(3)$ | 2 | $z3, b13$ | $\circ$ |
| 164 | $\mathfrak{A}_{11}$ | 2 | | $-$ |
| 164 | $\mathfrak{A}_{12}$ | 2 | | $-$ |
| 165 | $\mathfrak{A}_{11}$ | 0, 11 | | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 165 | $\mathfrak{A}_{12}$ | $\neq 2, 3$ | | $+$ |
| 165 | $\mathfrak{A}_{13}$ | 13 | | $+$ |
| 165 | $U_5(2)$ | $\neq 2, 3$ | | $+$ |
| 167 | $S_6(3)$ | 13 | | $+$ |
| 167 | $O_7(3)$ | 13 | | $+$ |
| 167 | $G_2(3)$ | 13 | | $+$ |
| 168 | $\mathfrak{A}_9$ | 0, 7 | | $+$ |
| 168 | $2.\mathfrak{A}_9$ | 5 | | $+$ |
| 168 | $2.\mathfrak{A}_9$ | 0, 7 | $i15$ | $\circ$ |
| 168 | $2.\mathfrak{A}_{10}$ | 5 | $r21$ | $+$ |
| 168 | $S_6(2)$ | $\neq 2, 3$ | | $+$ |
| 168 | $2.S_6(2)$ | $\neq 2, 3$ | | $+$ |
| 168 | $S_6(3)$ | $\neq 3, 13$ | | $+$ |
| 168 | $O_7(3)$ | 0, 5, 7 | | $+$ |
| 168 | $2.O_8^+(2)$ | 5 | | $+$ |
| 168 | $G_2(3)$ | 0, 7 | | $+$ |
| 169 | $\mathfrak{A}_{20}$ | 19 | | $+$ |
| 169 | $\mathfrak{A}_{21}$ | 19 | | $+$ |
| 170 | $\mathfrak{A}_{20}$ | $\neq 2, 3, 19$ | | $+$ |
| 170 | $U_9(2)$ | $\neq 2$ | | $-$ |
| 171 | $\mathfrak{A}_{20}$ | $\neq 2, 5,$ | | $+$ |
| 171 | $\mathfrak{A}_{21}$ | 3, 7 | | $+$ |
| 171 | $3.U_9(2)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 171 | $S_6(7)$ | $\neq 7$ | $b7$ | $\circ$ |
| 171 | $O_8^-(2)$ | 7 | | $+$ |
| 171 | $3.J_3$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 171 | $3.J_3$ | 0, 17, 19 | $z3, b5$ | $\circ$ |
| 172 | $2.S_6(7)$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 174 | $S_4(7)$ | 2 | | $+$ |
| 174 | $6.M_{22}$ | 11 | $z12$ | $\circ$ |
| 174 | HS | 11 | | $+$ |
| 175 | $S_4(7)$ | $\neq 2, 7$ | | $+$ |
| 175 | $O_8^+(2)$ | $\neq 2, 3$ | | $+$ |
| 175 | $J_2$ | $\neq 2, 3$ | | $+$ |
| 175 | HS | 0, 5, 7 | | $+$ |
| 176 | $U_5(2)$ | $\neq 2, 3$ | | $+$ |
| 176 | $2.U_6(2)$ | $\neq 2, 3$ | | $+$ |
| 176 | $M_{12}$ | 0, 11 | | $+$ |
| 176 | $4.M_{22}$ | 0, 11 | $i1$ | $\circ$ |
| 176 | 2.HS | $\neq 2, 5$ | $i1$ | $\circ$ |
| 176 | $2.Fi_{22}$ | 3 | $b13$ | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 180 | $S_4(19)$ | 2 | $b19$ | ∘ |
| 180 | $2.S_4(19)$ | $\neq 2, 19$ | $b19$ | ∘ |
| 181 | $L_3(13)$ | 3, 61 | | + |
| 181 | $S_4(19)$ | $\neq 2, 19$ | $b19$ | ∘ |
| 182 | $L_3(13)$ | 0, 2, 7 | | + |
| 182 | $U_6(3)$ | $\neq 3$ | | − |
| 182 | $2.U_6(3)$ | $\neq 2, 3$ | $i1$ | ∘ |
| 182 | $2.S_6(3)$ | $\neq 2, 3$ | | − |
| 182 | $2.S_6(3)$ | $\neq 2, 3$ | $z3$ | ∘ |
| 182 | $O_7(3)$ | $\neq 2, 3$ | | + |
| 182 | $G_2(3)$ | $\neq 2, 3$ | | + |
| 183 | $L_3(13)$ | $\neq 2, 13$ | | + |
| 183 | $L_3(13)$ | $\neq 2, 13$ | $i1$ | ∘ |
| 183 | $3.L_3(13)$ | 0, 7, 61 | $z12$ | ∘ |
| 183 | $3.L_3(13)$ | $\neq 3, 13$ | $z3$ | ∘ |
| 183 | $U_6(3)$ | $\neq 2, 3$ | | + |
| 185 | $O_{10}^+(2)$ | 3, 17 | | + |
| 186 | $\mathfrak{A}_{11}$ | 2 | | + |
| 186 | $L_3(5)$ | $\neq 2, 5$ | | + |
| 186 | $S_{10}(2)$ | 3 | | + |
| 186 | $O_{10}^+(2)$ | $\neq 2, 3, 17$ | | + |
| 186 | $O_{10}^-(2)$ | 3 | | + |
| 187 | $S_{10}(2)$ | $\neq 2, 3$ | | + |
| 187 | $O_{10}^-(2)$ | $\neq 2, 3$ | | + |
| 188 | $\mathfrak{A}_{11}$ | 5 | | + |
| 188 | $\mathfrak{A}_{21}$ | 2, 5 | | + |
| 188 | $\mathfrak{A}_{22}$ | 2, 5 | | + |
| 188 | $U_4(3)$ | 5 | | + |
| 189 | $\mathfrak{A}_9$ | $\neq 2, 5$ | | + |
| 189 | $\mathfrak{A}_{21}$ | $\neq 2, 5, 19$ | | + |
| 189 | $L_4(4)$ | 5 | | + |
| 189 | $L_4(4)$ | $\neq 2, 5$ | $b5$ | + |
| 189 | $3.U_3(8)$ | $\neq 2, 3$ | $z3$ | ∘ |
| 189 | $U_4(3)$ | 0, 7 | | + |
| 189 | $3_2.U_4(3)$ | $\neq 3, 5$ | $z3$ | ∘ |
| 189 | $S_6(2)$ | $\neq 2, 5$ | | + |
| 189 | $3.G_2(3)$ | 13 | $z3$ | ∘ |
| 189 | $3.G_2(3)$ | 0, 7 | $z3$, $b13$ | ∘ |
| 189 | $J_2$ | 5 | | + |
| 189 | $J_2$ | $\neq 2, 5$ | $b5$ | + |
| 190 | $\mathfrak{A}_{21}$ | $\neq 2, 3, 7$ | | + |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 190 | $\mathfrak{A}_{22}$ | 11 | | $+$ |
| 190 | $M_{22}$ | 11 | | $+$ |
| 190 | $2.J_2$ | 5 | | $-$ |
| 194 | $S_6(3)$ | 7 | | $+$ |
| 194 | $O_7(3)$ | 7 | | $+$ |
| 195 | $S_6(3)$ | 0, 5, 13 | | $+$ |
| 195 | $O_7(3)$ | 0, 5, 13 | | $+$ |
| 196 | $\mathfrak{A}_{13}$ | 5 | | $+$ |
| 196 | $\mathfrak{A}_{14}$ | 5 | | $+$ |
| 196 | $S_4(8)$ | $\neq 2$ | | $+$ |
| 196 | $S_6(2)$ | 3 | | $+$ |
| 196 | ${}^3D_4(2)$ | $\neq 2$ | | $+$ |
| 198 | $\mathfrak{A}_{10}$ | 2 | | $+$ |
| 198 | $\mathfrak{A}_{11}$ | 2 | | $+$ |
| 199 | $\mathfrak{A}_{10}$ | 7 | | $+$ |
| 199 | $\mathfrak{A}_{11}$ | 7 | | $+$ |
| 199 | $J_2$ | 7 | | $+$ |
| 200 | $\mathfrak{A}_{10}$ | 2 | | $+$ |
| 200 | $2.S_4(7)$ | $\neq 2, 7$ | $b7$ | $\circ$ |
| 201 | $S_6(2)$ | 7 | | $+$ |
| 202 | $2.J_2$ | 5 | | $-$ |
| 203 | $S_8(2)$ | 3 | | $+$ |
| 203 | $O_8^-(2)$ | 3, 5 | | $+$ |
| 204 | $3_1.U_4(3)$ | 2 | $z3$ | $\circ$ |
| 204 | $U_5(4)$ | $\neq 2$ | | $\circ$ |
| 204 | $S_4(4)$ | $\neq 2, 5$ | $b5$ | $+$ |
| 204 | $O_8^-(2)$ | $\neq 2, 3$ | | $+$ |
| 205 | $5.U_5(4)$ | $\neq 5$ | $z5$ | $\circ$ |
| 207 | $\mathfrak{A}_{13}$ | 11 | | $+$ |
| 207 | $S_4(4)$ | 17 | | $+$ |
| 208 | $\mathfrak{A}_{13}$ | $\neq 3, 5, 11$ | | $+$ |
| 208 | $\mathfrak{A}_{14}$ | 2 | | $+$ |
| 208 | $\mathfrak{A}_{22}$ | 3, 7 | | $+$ |
| 208 | $\mathfrak{A}_{23}$ | 3, 7 | | $+$ |
| 208 | $L_4(3)$ | 2, 5 | | $+$ |
| 208 | $2.L_4(3)$ | $\neq 2, 3$ | $i2$ | $\circ$ |
| 208 | $S_4(5)$ | $\neq 5$ | $b5$ | $+$ |
| 208 | $2.S_4(5)$ | $\neq 2, 5$ | $b5$ | $-$ |
| 208 | $M_{23}$ | 7 | | $+$ |
| 208 | $2.Suz$ | 3 | | $-$ |
| 209 | $\mathfrak{A}_{22}$ | $\neq 2, 3, 5, 7$ | | $+$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

Continued from the previous page

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 209 | $J_1$ | 0, 11, 19 | | $+$ |
| 210 | $\mathfrak{A}_{10}$ | 0, 7 | | $+$ |
| 210 | $\mathfrak{A}_{11}$ | $\neq 2, 11$ | | $+$ |
| 210 | $\mathfrak{A}_{12}$ | 3 | | $+$ |
| 210 | $\mathfrak{A}_{22}$ | $\neq 2, 11$ | | $+$ |
| 210 | $\mathfrak{A}_{23}$ | 23 | | $+$ |
| 210 | $U_4(3)$ | $\neq 2, 3$ | | $+$ |
| 210 | $2.U_4(3)$ | $\neq 2, 3$ | $i1$ | $\circ$ |
| 210 | $3_1.U_4(3)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 210 | $6_1.U_4(3)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 210 | $U_6(2)$ | 3 | | $+$ |
| 210 | $3.U_6(2)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 210 | $S_6(2)$ | $\neq 2, 3$ | | $+$ |
| 210 | $O_8^+(2)$ | $\neq 2, 3$ | | $+$ |
| 210 | $M_{22}$ | $\neq 2, 11$ | | $+$ |
| 210 | $2.M_{22}$ | $\neq 2, 11$ | | $+$ |
| 210 | $3.M_{22}$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 210 | $6.M_{22}$ | 0, 5, 7 | $z3$ | $\circ$ |
| 210 | $6.M_{22}$ | $\neq 2, 3$ | $z12$ | $\circ$ |
| 210 | $M_{23}$ | 23 | | $+$ |
| 210 | HS | 5 | | $+$ |
| 210 | McL | 3, 5 | | $+$ |
| 214 | $J_3$ | 19 | $b17$, $y9$ | $+$ |
| 216 | $\mathfrak{A}_9$ | 0 | | $+$ |
| 216 | $2.\mathfrak{A}_{10}$ | 0, 3 | | $+$ |
| 216 | $12_1.U_4(3)$ | 0, 7 | $z12$ | $\circ$ |
| 216 | $12_2.U_4(3)$ | $\neq 2, 3$ | $z12$ | $\circ$ |
| 216 | $S_6(2)$ | 0 | | $+$ |
| 216 | $2.J_2$ | 0, 3 | | $-$ |
| 217 | $\mathfrak{A}_{10}$ | 5 | | $+$ |
| 217 | $L_5(2)$ | $\neq 2$ | | $+$ |
| 217 | $L_6(2)$ | $\neq 2$ | | $+$ |
| 218 | ${}^3D_4(3)$ | 2 | | $+$ |
| 218 | ${}^3D_4(3)$ | 73 | | $+$ |
| 219 | ${}^3D_4(3)$ | $\neq 2, 3, 73$ | | $+$ |
| 220 | $\mathfrak{A}_{13}$ | $\neq 2, 13$ | | $+$ |
| 220 | $\mathfrak{A}_{14}$ | 7 | | $+$ |
| 220 | $U_4(4)$ | 5 | | $+$ |
| 220 | $U_5(2)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 220 | $M_{23}$ | 2 | $b7$, $b23$ | $\circ$ |
| 220 | $M_{24}$ | 2 | $b7$, $b23$ | $\circ$ |

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Table 3: Absolutely irreducible representations of quasi-simple groups

Continued from the previous page

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 220 | 2.Suz | $\neq 2, 3$ | | $-$ |
| 221 | $\mathfrak{A}_{12}$ | 7 | | + |
| 221 | $\mathfrak{A}_{13}$ | 7 | | + |
| 221 | $U_4(4)$ | $\neq 2$ | $b5$ | + |
| 223 | $S_4(7)$ | 5 | | + |
| 224 | $2.\mathfrak{A}_9$ | 0, 7 | | + |
| 224 | $\mathfrak{A}_{10}$ | $\neq 2, 5$ | | + |
| 224 | $4.U_4(3)$ | $\neq 2, 3$ | $z12$ | $\circ$ |
| 224 | $S_4(7)$ | $\neq 2, 5, 7$ | | + |
| 224 | $2.O_8^+(2)$ | $\neq 2, 5$ | | + |
| 224 | $J_2$ | 0, 7 | $b5$ | + |
| 225 | $\mathfrak{A}_{10}$ | 0, 5 | | + |
| 225 | $S_4(4)$ | $\neq 2, 17$ | $d17$ | + |
| 225 | $J_2$ | $\neq 2, 7$ | | + |
| 229 | $\mathfrak{A}_{23}$ | 11 | | + |
| 229 | $\mathfrak{A}_{24}$ | 11 | | + |
| 229 | $U_6(2)$ | 3 | | + |
| 229 | $M_{23}$ | 11 | | + |
| 229 | $M_{24}$ | 11 | | + |
| 230 | $\mathfrak{A}_{23}$ | $\neq 3, 7, 11$ | | + |
| 230 | $\mathfrak{A}_{24}$ | 2 | | + |
| 230 | $M_{23}$ | 0, 5, 23 | | + |
| 230 | McL | 2, 5 | | + |
| 230 | $Co_3$ | 2, 5 | | + |
| 230 | $Co_2$ | 2 | | + |
| 231 | $\mathfrak{A}_{11}$ | 0, 7, 11 | | + |
| 231 | $\mathfrak{A}_{23}$ | $\neq 2, 23$ | | + |
| 231 | $\mathfrak{A}_{24}$ | 3 | | + |
| 231 | $U_6(2)$ | $\neq 2, 3$ | | + |
| 231 | $3.U_6(2)$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 231 | $M_{22}$ | $\neq 2, 5$ | | + |
| 231 | $3.M_{22}$ | $\neq 2, 3$ | $z3$ | $\circ$ |
| 231 | $M_{23}$ | $\neq 2, 23$ | | + |
| 231 | $M_{23}$ | $\neq 2, 3, 5$ | $i15$ | $\circ$ |
| 231 | $M_{24}$ | 3, 5 | | + |
| 231 | $M_{24}$ | $\neq 2, 3, 5$ | $i15$ | $\circ$ |
| 231 | HS | $\neq 2, 5$ | | + |
| 231 | McL | 0, 7, 11 | | + |
| 231 | $Co_3$ | 3 | | + |
| 233 | $\mathfrak{A}_{13}$ | 5 | | + |
| 233 | $\mathfrak{A}_{14}$ | 5 | | + |

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Table 3: Absolutely irreducible representations of quasi-simple groups

*Continued from the previous page*

| $d$ | $G$ | $\ell$ | field | ind |
|---|---|---|---|---|
| 234 | $L_4(3)$ | 0, 13 | | + |
| 236 | $2.J_2$ | 3 | | − |
| 238 | $S_8(2)$ | $\neq 2, 3$ | | + |
| 240 | $U_3(16)$ | $\neq 2$ | | − |
| 241 | $U_3(16)$ | $\neq 2, 17$ | $z17$ | ∘ |
| 244 | $J_3$ | 2 | $b17$ | + |
| 245 | $O_8^-(3)$ | 13 | | + |
| 246 | $O_8^-(3)$ | $\neq 3, 13$ | | + |
| 246 | ${}^2F_4(2)'$ | 2 | | + |
| 246 | He | 2 | $b17$ | + |
| 248 | $S_4(5)$ | 2 | $b5$ | + |
| 248 | Th | all | | + |

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