

Selected Solutions to X4

- ① Note that $f_x(a,b)$, etc. are constant.
Then compute:

$$Q_x(x,y) = f_x(a,b) + f_{xx}(a,b)(x-a) + f_{xy}(a,b)(y-b)$$

Setting $x=a$, $y=b$ yields $Q_x(a,b) = f_x(a,b)$.
Similarly for $Q_y(a,b)$, $Q_{xx}(a,b)$, etc.

- ④ b) apply change of coordinates

$$x' = \frac{1}{\sqrt{a}}x + \frac{1}{2}b\sqrt{a}y, \quad y' = \frac{\sqrt{b}}{2\sqrt{a}}y$$

Then $f(x,y) = f(x',y') = (x')^2 + (y')^2$
has a local minimum at $(0,0)$
because $f(0,0) = 0$ and $f(x,y) \geq 0$
for all (x,y) .

Similar arguments solve c) & d)
like in the lecture.