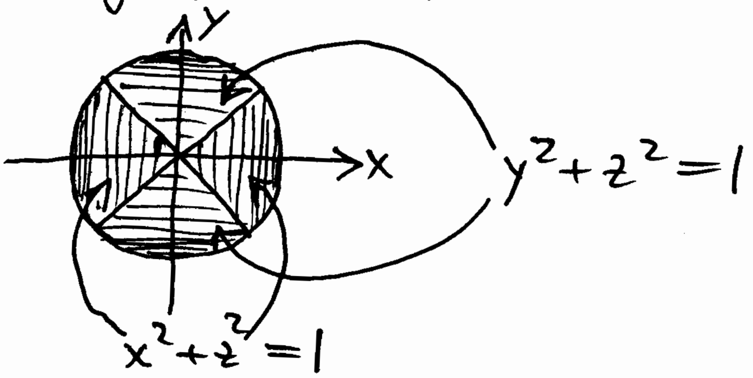
Solution X 6

Looking from a point on the z-axis we see



Note that the projection to the x-y-plane of the intersection curves of $x^2+z^2=1$ and $y^2+z^2=1$ are two diagonal lines:

$$x^2+z^2=1=y^2+z^2 \Longrightarrow 0=x^2-y^2=(x-y)(x+y)$$

$$\Longleftrightarrow \; y=x \;\underline{\text{or}}\; y=-x.$$

The volume is

$$V=8\int_{-\pi/4}^{\pi/4}\int_0^{1}\int_0^{\sqrt{1-x^2}} 1\,dz\,r\,dr\,d\theta = 8\int_{-\pi/4}^{\pi/4}\int_0^{1}\sqrt{1-r^2\cos^2\theta}\,r\,dr\,d\theta$$

$$=8\int_{-\pi/4}^{\pi/4}\left[-\frac{1}{2\cos^2\theta}\,\frac{2}{3}\sqrt{1-r^2\cos^2\theta}^{\,3}\right]_0^1 d\theta$$

$$=-\frac{8}{3}\int_{-\pi/4}^{\pi/4}\underbrace{\frac{\sin^3\theta}{\cos^2\theta}}_{\substack{\text{antisymm.}\\ \Rightarrow\text{ integral}=0}}-\frac{1}{\cos^2\theta}\,d\theta=\frac{8}{3}\int_{-\pi/4}^{\pi/4}\frac{1}{\cos^2\theta}\,d\theta$$

$$=\frac{8}{3}\Big[\tan\theta\Big]_{-\pi/4}^{\pi/4}=\frac{8}{3}(1-(-1))=\frac{16}{3}.$$