First Name: ______________________ **Last Name:** ______________________

OSU Number: ____________________ **Signature:** ___________________________

OKLAHOMA STATE UNIVERSITY
Department of Mathematics

MATH 2163 (Calculus III)
Instructor: Dr. Mathias Schulze

MIDTERM 2
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Duration: 50 minutes

No aids allowed.

This examination paper consists of **4** pages and **5** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer 5 questions.

To obtain credit, you must give arguments to support your answers.

For graders' use:

	Score
1 (9)	
2 (9)	
3 (6)	
4 (6)	
5 (9)	
Total (39)	

1. [9] Consider the function $f(p,q) = qe^{-p} + pe^{-q}$ at the point $P(0,0)$.

   (a) Find the maximum rate of change of f at P and the direction in which it occurs.

   (b) Find the directional derivative of f at P in direction of the x-axis.

   (c) Find the equation of the tangent and normal lines to the curve $f(p,q) = 0$ at P.

Solution:

(a) Since $\nabla f(0,0) = \langle -qe^{-p} + e^{-q}, e^{-p} - pe^{-q} \rangle (0,0) = \langle 1, 1 \rangle$, the maximum rate of change is $\sqrt{2}$ and occurs in direction $45°$.

(b) $D_{(1,0)} f(0,0) = \langle 1, 1 \rangle \bullet \langle 1, 0 \rangle = 1$

(c) The tangent line is given by $x + y = 0$, the normal line by $x - y = 0$.

2. [9] Give an example of a function with the given property.

   (a) a local maximum at $(1,1)$

   (b) a saddle point at $(1,1)$

   (c) a local minimum at $(0,0)$ that can not be detected using the 2nd derivative test.

Solution:

(a) $f(x,y) = -(x-1)^2 - (y-1)^2$ (or even $f(x,y) = 0$ works)

(b) $f(x,y) = (x-1)^2 - (y-1)^2$

(c) $f(x,y) = x^2 + y^4$ (or even $f(x,y) = 0$ works)

3. [6] Consider the function $f(x,y) = xy + \frac{1}{x} + \frac{1}{y}$.

   (a) Find all critical points and determine whether they are local maxima, local minimal, or saddle points.

   (b) Are there any global maxima or minima? Explain why.

Solution:

(a) Setting $\nabla f = \langle y - 1/x^2, x - 1/y^2 \rangle = 0$ gives $x = y = 1$. The Hessian at this point is $\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$, so it is a local minimum by the 2nd derivative test.

(b) Since $\lim_{x \to 0\pm} f(x,1) = \pm\infty$, there is no global maximum or minimum.

4. [6] Find the maximum and minimum values of the function $f(x,y) = 2x^2 + 3y^2 - 4x - 5$ subject to the constraint $x^2 + y^2 \leq 16$.

Solution: To find critcal points (x,y) with $x^2 + y^2 < 16$, set $\nabla f = \langle 4x - 4, 6y \rangle = 0$. This gives $x = 1$ and $y = 0$, and $f(1,0) = -7$. To find critcal points with $g = x^2 + y^2 - 16 = 0$, apply Lagrange multipliers: Using $\nabla g = \langle 2x, 2y \rangle$, the system of equations $\nabla f = \lambda \nabla g$, $g = 0$, becomes: $2x - 2 = \lambda x$, $3y = \lambda y$, $x^2 + y^2 = 16$. Rewrite the first two equations as $(2 - \lambda)x = 2$, $(3 - \lambda)y = 0$. If $y = 0$ then $x = \pm 4$, and the corresponding values of f are 11 and 43. Otherwise, $\lambda = 3$, $x = -2$, $y = \pm 2\sqrt{3}$, and the value of f is 47. Thus, f has minimum value -7 at $(1,0)$, and maximum value 47 at $(-2, \pm 2\sqrt{3})$.

5. [9]

(a) Evaluate the integral by changing the order of integration: $\int_0^1 \int_x^1 e^{x/y} dydx$.

(b) Compute the average of $f(x, y) = xy$ over the triangle Δ with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$.

(c) Compute the volume V under the surface $z = 2x + y^2$, and above the region bounded by $x = y^2$, $x = y^3$.

Solution:

(a) $\int_0^1 \int_x^1 e^{x/y} dydx = \iint_{\{(x,y)|0\leq x\leq 1, x\leq y\leq 1\}} e^{x/y} dA = \iint_{\{(x,y)|0\leq y\leq 1, 0\leq x\leq y\}} e^{x/y} dA = \int_0^1 \int_0^y e^{x/y} dxdy = \int_0^1 [ye^{x/y}]_{x=0}^y dy = (e - 1) \int_0^1 ydy = \frac{1}{2}(e - 1)$.

(b) $f_{\text{av}} = \frac{1}{A(\Delta)} \iint_\Delta xydA = 2 \int_0^1 \int_0^x xydydx = \int_0^1 [xy^2]_{y=0}^x dx = \int_0^1 x^3 dx = \frac{1}{4}$

(c) $V = \int_0^1 \int_{y^3}^{y^2} \int_0^{2x+y^2} dzdydx = \int_0^1 \int_{y^3}^{y^2} 2x + y^2 dxdy = \int_0^1 [x^2 + xy^2]_{x=y^3}^{y^2} dy = \int_0^1 2y^4 - y^5 - y^6 dy = \frac{2}{5} - \frac{1}{6} - \frac{1}{7}$

End of examination
Total pages: 4
Total marks: 39