**1.7.3** Let $\ell = \sup(S)$ and pick $\varepsilon > 0$ arbitrary. Then $\ell - \varepsilon < s \leq \ell$ for some $s \in S$ by definition of $\ell$. Therefore $s \in (\ell - \varepsilon, \ell] \subset B(\ell, \varepsilon)$ and $S \cap B(\ell, \varepsilon) \neq \emptyset$. It follows that $\ell \in \overline{S}$.

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**1.7.4** Let $L = \operatorname{diam}(A)$ and $M = \operatorname{diam}(B)$, and pick $\varepsilon > 0$ arbitrary. Then there are $p, q \in A$ such that $L - \varepsilon < |p - q| \leq L$, by def. of $L$. By def. of $M$, we have $|p-q| \leq M$ since $p, q \in A \subset B$. We conclude that $L - \varepsilon < |p-q| \leq M$ for all $\varepsilon > 0$. Thus, $L \leq M$.

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**1.7.7** No. Counter-example: $A = \{(x,y) \mid y \leq 0\}$, $B = \{(x,y) \mid y \geq \frac{1}{x^2}\}$
(Closedness of $A, B$ will follow from continuity of $y$ and $x^2 y$ later.)
Let $p_n = (n, 0) \in A$, $q_n = (n, \frac{1}{n^2}) \in B$. Then $|p_n - q_n| = \frac{1}{n^2} < \varepsilon$ for any $\varepsilon > 0$ and suitably chosen $n \in \mathbb{N}$. Thus, $\operatorname{dist}(A, B) = 0$.

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**1.7.10** Let $\{p_n\}$ be bounded w/ unique limit point $p$. Let $\varepsilon > 0$, and assume there are infinitely many $n \in \mathbb{N}$ s.th. $p_n \notin B(p, \varepsilon)$. Then there is a subsequence $q_n = p_{\alpha(n)} \in B(p, \varepsilon)^c$.

As such, $\{q_n\}$ is bounded and has a limit point $q$ which is then also a limit point of $\{p_n\}$. As $B(p, \varepsilon)$ is open, $q$ must be in $B(p, \varepsilon)^c$. Then $p \neq q$ contradicts the hypothesis of a unique limit point. Therefore, for all but finitely many $n \in \mathbb{N}$, we have $p_n \in B(p, \varepsilon)$. If $n_1, \dots, n_k$ are the indices w/ $p_n \notin B(p, \varepsilon)$, set $N = \max\{n_1, \dots, n_k\}$. Then $p_n \in B(p, \varepsilon)$ and hence $|p_n - p| < \varepsilon$ for all $n \geq N$.