2.2.2| <u>Claim</u>: $Q(x,y) = x/y$ is continuous.

<u>Proof</u>: Fix $p_0 = (x_0, y_0) \in D_Q$; then $y_0 \neq 0$. For given $\delta > 0$, pick $\varepsilon < \min\{|y_0|/2, \frac{\delta}{4}\frac{|y_0|^2}{|p_0|}\}$. Then, for $p = (x,y)$ s.th. $|p - p_0| < \varepsilon$, we have

$$|y_0| - |y| \le |y - y_0| \le |p - p_0| < \varepsilon < \frac{|y_0|}{2} \Rightarrow |y| > \frac{|y_0|}{2},$$

and hence

$$|Q(p) - Q(p_0)| = \left|\frac{y_0(x-x_0) + x_0(y_0 - y)}{y y_0}\right|$$
$$\le \frac{2}{|y_0|^2}\left(|y_0| - |x_0|\right)|p - p_0| < \frac{4|p_0|}{|y_0|^2}\cdot\varepsilon < \delta$$

2.2.4| f is continuous, since, for $p = (x,y)$ s.th. $|p| < 1$, we have

$$|f(p) - f(0)| = \frac{|x|^2|y|^2}{|p|^2} \le \frac{|p|^2|p|^2}{|p|^2} = |p|^2 < |p| = |p - 0|$$

~~2.2.5| $(b,c) = B\left(\frac{b+c}{2}, \frac{c-b}{2}\right)$ is open, and therefore $S = f^{-1}((b,c))$ is open, by definition of continuity.~~

2.2.5| Since $(b,c) = B\left(\frac{b+c}{2}, \frac{c-b}{2}\right)$ and $B(x,\varepsilon) = (x-\varepsilon, x+\varepsilon)$, the open balls in $\mathbb{R}$ are exactly the open intervals. Any open set in $\mathbb{R}$ is an infinite union of open intervals and preimage commutes with unions. Therefore continuity of $\mathbb{R}^n \supset D \xrightarrow{f} \mathbb{R}$ is equivalent to

$$f^{-1}((b,c)) = \{p \in D \mid b < f(p) < c\} \subset D$$

being open for all $b < c$.