

2.4.3 | a) F polyn. $\Rightarrow F$ cont. $\Rightarrow A = F^{-1}([0, \infty))$ closed,

$C = F^{-1}((0, \infty))$ open. $\Rightarrow \bar{A} = A, C \subset \overset{\circ}{A}$.

$\Rightarrow \partial A = \bar{A} \setminus \overset{\circ}{A} \subset A \setminus C = F^{-1}(0) = B$.

b) Let $F(x, y) = x^2 y$, then $p = (0, 1) \in B \setminus \partial A$. Proof:

Let $q = (a, b) \in B(p, 1) \Rightarrow a^2 + (b-1)^2 < 1 \Leftrightarrow a^2 + b^2 = 2b$

$\Rightarrow b > 0 \Rightarrow F(q) \geq 0$. Thus, $B(p, 1) \subset A \Rightarrow p \in \overset{\circ}{A} \Rightarrow p \in \partial A$

2.4.5 | Let $x \in [0, 1]$. Since $\mathbb{Q} = \mathbb{R}, \exists \{x_n\} \subset [0, 1] \cap \mathbb{Q}$:

$x_n \rightarrow x$. By continuity, $f(x) = \lim_{n \rightarrow \infty} f(x_n) = \lim_{n \rightarrow \infty} g(x_n) = g(x)$

2.4.8 | Let S be connected, $f: S \rightarrow \mathbb{R}$ locally const,

i.e. $\forall a \in \mathbb{R}: f^{-1}(a) \subset S$ open. For $p \in S$ set

$U = f^{-1}(f(p)) \neq \emptyset, V = f^{-1}(\mathbb{R} \setminus \{f(p)\})$. Then

$U, V \subset S$ open, $U \neq \emptyset$, and $S = U \dot{\cup} V$.

By connectedness of $S, V = \emptyset$ and $f = f(p)$.

2.4.9 | If $S = U \dot{\cup} V$ is disconn., $f: S \rightarrow \mathbb{R}$ def.

by $f(p) = \begin{cases} 2, & p \in U \\ 3, & p \in V \end{cases}$ is cont. Conversely,

if such an f exists, setting $U = f^{-1}(2), V = f^{-1}(3)$, yields disconn. of S .