

3.2.1) By 3.2.3, $f: (a, b) \rightarrow \mathbb{R}$ is continuous, so the MVT applies to any interval $[x, y] \subset (a, b)$ and gives an $z \in (x, y)$ s.t. $f(x) - f(y) = f'(z)(x - y) = 0$. Thus, $f(x) = f(y)$ for all $x, y \in (a, b)$ and f is constant.

3.2.3) Fix $x_0 \in (a, b)$. Then

$$\lim_{x \rightarrow x_0} (f(x) - f(x_0)) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} \cdot \lim_{x \rightarrow x_0} (x - x_0) = f'(x_0) \cdot 0 = 0$$

and hence f is continuous at x_0 .

3.2.5) Note that some of the zeros of f and g may coincide. But if x is a zero of f and g then it is also a zero of $F' = f'g + f g'$.

Combining this observation w/ Cor. 1, we conclude that F' has at least 7 zeros. Applying Cor. 2 once more shows that $F^{(2)}$ has at least 6 zeros.

3.2.9) Let M be a bound for f' . Then by 3.2.3 and the MVT, $|f(a) - f(b)| \leq M|a - b|$. So f is Lipschitz-continuous and hence uniformly continuous. Indeed, for $\varepsilon > 0$ we can pick $\delta = \frac{\varepsilon}{M}$. Then $\forall a, b$ s.t. $|a - b| < \delta$ we have $|f(a) - f(b)| \leq M \cdot \delta = \varepsilon$.
