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OKLAHOMA STATE UNIVERSITY
Department of Mathematics

MATH 2144 (Calculus I)
Instructor: Dr. Mathias Schulze

MIDTERM 2
October 27, 2010

Duration: 50 minutes

No aids allowed.

This examination paper consists of **7** pages and **10** questions. Please bring any discrepancy to the attention of an invigilator. The number in brackets at the start of each question is the number of points the question is worth.

Answer **8** out of **10** questions.

Skip (or mark) two problems. Give arguments to support your answers.

For graders' use:

	Score
1 (5)	
2 (5)	
3 (5)	
4 (5)	
5 (5)	
6 (5)	
7 (5)	
8 (5)	
9 (5)	
10 (5)	
Total (50)	

1. [5] Find $f'(x)$ and $f''(x)$ where $f(x) = \frac{x}{3+e^x}$.

   Solution:

$$\begin{aligned} f'(x) &= \frac{3 + e^x - xe^x}{(3 + e^x)^2} \\ f''(x) &= \frac{-xe^x(3 + e^x)^2 - 2e^x(3 + e^x)(3 + e^x - xe^x)}{(3 + e^x)^4} \\ &= \frac{-xe^x(3 + e^x) - 2e^x(3 + e^x - xe^x)}{(3 + e^x)^3} \\ &= \frac{xe^x - 2e^x - 3x - 6}{(3 + e^x)^3} e^x \end{aligned}$$

2. [5] Differentiate $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.

   Solution:

$$\begin{aligned} y &= \tanh(x) \\ y' &= \operatorname{sech}^2(x) \end{aligned}$$

3. [5] Find the 1000th derivative of $f(x) = xe^{-x}$.

Solution: Since $((x-k)e^{-x})' = -(x-(k+1))e^{-x}$, we have $f^{(k)} = (-1)^k(x-k)e^{-x}$, and hence $f^{(1000)} = (x-1000)e^{-x}$.

4. [5] Find y' by implicit differentiation: $e^{\frac{x}{y}} = x - y$.

Solution:

$$\begin{aligned} e^{\frac{x}{y}}\left(\frac{1}{y} - \frac{xy'}{y^2}\right) &= 1 - y' \\ \left(1 - \frac{x}{y^2}e^{\frac{x}{y}}\right)y' &= 1 - \frac{1}{y}e^{\frac{x}{y}} \\ y' &= y\frac{y - e^{\frac{x}{y}}}{y^2 - xe^{\frac{x}{y}}} \end{aligned}$$

5. [5] Use implicit differentiation to find the equation of the tangent line to the curve $x^2 + xy + y^2 = 3$ at the point $(1, 1)$.

Solution: Differentiation gives

$$2x + y + xy' + 2yy' = 0.$$

Setting $m = y'(1, 1)$ and substituting $(x, y) = (1, 1)$, this becomes

$$2 + 1 + m + 2m = 0.$$

So $m = -1$ and the equation of the tangent line at $(1, 1)$ reads

$$y = 1 + m(x - 1) = -x + 2.$$

6. [5] Differentiate the function $y = \log_5(xe^x)$.

Solution:

$$y' = \frac{(x+1)e^x}{\ln(5)xe^x} = \left(1 + \frac{1}{x}\right) \ln\frac{1}{5}$$

7. [5] Use logarithmic differentiation to differentiate $y = x^{\sin x}$.

Solution: Applying ln gives

$$\ln y = \sin(x)\ln(x).$$

Then differentiate to obtain

$$\frac{y'}{y} = \cos(x)\ln(x) + \frac{\sin x}{x}$$

which yields

$$y' = x^{\sin(x)}\left(\cos(x)\ln(x) + \frac{\sin x}{x}\right).$$

8. [5] If a snow ball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm.

Solution: The surface area of the snow ball is

$$A = 4\pi r^2 = \pi d^2$$

where $r = r(t)$ and $d = d(t)$ denote the radius and diameter, respectively. So, at the time $t = t_0$ when $d(t_0) = 10$, we have

$$-1 = A'(t_0) = 2\pi d(t_0)d'(t_0) = 20\pi d'(t_0)$$

and hence $d'(t_0) = -\frac{1}{20\pi}$.

9. [5] The radius of a circular disc is given as 10 cm with a maximum error in measurement of 0.1 cm. Use differentials to estimate the relative error in the calculated area of the disc.

Solution: From $A = \pi r^2$, we compute $dA = 2\pi r dr$, and then the relative error is

$$\frac{\Delta A}{A} \approx \frac{dA}{A} = 2\frac{dr}{r} = 0.02 = 2\%.$$

10. [5] Find the absolute maximum and minimum values of the function $f(x) = \frac{x^2-4}{x^2+4}$, where $-4 \leq x \leq 4$.

Solution: Since

$$f'(x) = \frac{16x}{(x^2+4)^2},$$

the critical numbers are $-4, 0, 4$. Therefore,

$$f(\pm 4) = \frac{3}{5} \text{ and } f(0) = -1$$

are the maximum and minimum values.

End of examination
Total pages: 7
Total marks: 50