7.2.6

"$\Rightarrow$" explained in class.

"$\Leftarrow$" Let $V \subseteq \mathbb{R}^m$ open. To show that $f^{-1}(V) = U$ is open, pick $p \in U$ and set $q = f(p) \in V$. As V is open, there is a $\varepsilon > 0$ s.th. $B_\varepsilon(q) \subseteq V$. Then $B_\delta(q_1) \times \dots \times B_\delta(q_m) \subseteq B_\varepsilon(q)$ for $\delta = \varepsilon/m$, and hence

$$p \in \underbrace{f_1^{-1}(B_\delta(q_1)) \cap \dots \cap f^{-1}(B_\delta(q_m))}_{(*)} = f^{-1}(B_\delta(q_1) \times \dots \times B_\delta(q_m))$$
$$\subseteq f^{-1}(B_\varepsilon(q))$$
$$\subseteq f^{-1}(V) = U$$

where $(*)$ is open by assumption. Thus U is open as claimed.