HOMEWORK 3

Define your terminology and explain notation. If you require a standard result, such as one of the Sylow theorems, then state it (or cite a result from your textbook) before you use it; otherwise give clear and complete proofs of your claims. The problems are of equal value, 6 points each. Partial solutions will be considered on their merits.

Exercise 1. Show that $\bar{\mathbb{F}}_p = \varinjlim \mathbb{F}_q$. (Hint: First explain how to interpret the right hand side. Then apply the existence/uniqueness of finite fields to show that it has the desired properties.)

Exercise 2. Show that:

(a) $\mathbb{F}_{p^{nd}}/\mathbb{F}_{p^n}$ is normal and separable.

(b) $\mathrm{Aut}(\mathbb{F}_{p^{nd}}/\mathbb{F}_{p^n}) = \langle \phi^n \rangle \cong Z_d$ where ϕ is the Frobenius automorphism.

Exercise 3.

(a) If E/K is separable and F/K is purely inseparable then show that $[EF : F] = [E : K] = [EF : K]^{\mathrm{sep}}$ and $[EF : E] = [F : K] = [EF : K]^{\mathrm{insep}}$.

(b) Read Proposition V.6.11. Using the statement of this Proposition (without proof), write out the details of the proof of Corollary V.6.12.

Exercise 4. Let E be an algebraic extension of K. Show that every subring of E containing K is a field.

Exercise 5. Let K/F be an algebraic extension of fields of characteristic p. Suppose that for every $\alpha \in K$ there is an m such that $\alpha^{p^m} \in F$. Prove that K/F is purely inseparable.

Exercise 6. Let F be a field of characteristic p and let $\alpha \in F \setminus F^p$ where $F^p = \{x^p \mid x \in F\}$. Show that $X^{p^n} - \alpha$ is irreducible in $F[X]$ for all $n \geq 2$.

Final assignment will appear on Friday. Until then, problems might be changed, dropped or added.