

Seminar Cohen–Macaulay Rings

Winter Term 2013/14

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1 Outline of the talks

1.1 Completion

References: [AK70, §.II.1]

1.2 Associated Primes

References: [AK70, §.II.3]

1.3 Regular Sequences

References: [BH93, §1.1]

Topics:

- definition of regular sequence
- localization and completion
- exact sequence
- invariance under permutation (w/o proof?)

1.4 Depth and Ext

References: [BH93, §1.2], [AK70, §.III.3]

Topics:

- explain naive usage of Ext
- definition depth and grade
- depth in terms of Ext
- depth and exact sequences

1.5 Depth, Dimension, Extensions

References: [BH93, §1.2], [AK70, §.III.3]

Topics:

- depth and dimension of associated primes
- depth and flat extensions (w/o proof?)
- depth and finite extensions

1.6 Depth and Projective Dimension

References: [AK70, §.III.5], [BH93, §1.3]

Topics:

- definition of injective/projective dimension
- injective/projective dimension and Ext
- projective dimension and Tor for local rings
- Auslander–Buchsbaum formula

1.7 Rank, Fitting ideals, and Exact Sequences

References: [BH93, §1.4]

Topics:

- rank and exact sequences
- rank and localization
- Fitting ideals
- Buchsbaum–Eisenbud characterization of exactness

1.8 Depth and Koszul Complex

References: [Eis95, §.17.1-3], [BH93, §1.4]

Topics:

- motivation by small examples
- general construction
- building from parts
- depth in terms of exactness of the Koszul complex

1.9 Cohen–Macaulay Rings and Modules

References: [BH93, §2.1]

Topics:

- definition of Cohen–Macaulay (CM)
- dim, depth, and regular sequences in CM rings (see [BH93, §2.1.Thm. 2.1.2])
- CM under quotients by regular sequences
- CM and depth
- CM and unmixedness
- CM and flat extensions (completion)
- CM and field extensions (w/o proof?)
- CM rings are universally catenary

1.10 Regular Rings

References: [BH93, §2.2], [AK70, §.III.4+5]

Topics:

- definition regular ring
- regularity and completion
- regular rings are domains
- regularity under quotients
- regularity criterion in terms of grading by max ideal
- regular local rings are CM
- regularity in terms of projective dimension
- regularity and localization
- CM of finite extensions of regular rings
- regularity under flat extensions
- regularity of polynomial and power series rings

1.11 Serre's Conditions and Normality

References: [AK70, §.VII.2], [BH93, §2.1+2+4]

Topics:

- definition of Serre's conditions
- Serre's conditions and ring properties (see [AK70, §.VII.2.Prop. 2.2])
- depth and reflexivity
- S_n under flat extensions
- definition of normality
- regular rings are normal domains
- Serre's normality criterion

1.12 Complete intersections

References: [BH93, §2.3], [AK70, §.III.4]

Topics:

- definition of complete intersection (CI) rings
- characterization of CI rings
- CI under quotient by regular sequence
- CI for polynomial and power series rings
- CI under field extensions
- Koszul algebra and CI
- Wiebe's theorem (w/o proof?)

References

- [AK70] Allen Altman and Steven Kleiman. *Introduction to Grothendieck duality theory*. Lecture Notes in Mathematics, Vol. 146. Springer-Verlag, Berlin, 1970.
- [BH93] Winfried Bruns and Jürgen Herzog. *Cohen-Macaulay rings*, volume 39 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1993.
- [Eis95] David Eisenbud. *Commutative algebra*, volume 150 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, 1995. With a view toward algebraic geometry.