

Computer Algebra

Winter Semester 2013 - Problem Set 6

Due December 10, 2013, 12:00

Problem 1: Let $R = \mathbb{Q}[x, y, z]/\langle x^2 + y^2 + z^2 \rangle$, $M = R^3/\langle (x, xy, xz) \rangle$ and let $N = R^2/\langle (1, y) \rangle$. Moreover, let $\varphi : M \rightarrow N$ be the R -module homomorphism given by the matrix

$$A = \begin{pmatrix} x^2 + 1 & y & z \\ yz & 1 & -y \end{pmatrix}$$

Using SINGULAR,

- compute $\text{Ker}(\varphi)$ without using the command `modulo`,
- compute $\text{Im}(\varphi) \cap \langle (x^2, y^2) \rangle$ without using the command `intersect`,
- compute $\text{Ann}_R(\text{Im}(\varphi))$.

Problem 2: Give an algorithm to obtain a minimal resolution in the case of local rings, i.e. local monomial ordering on the polynomial ring, from Schreyer's resolution (see Algorithm 2.5.16 in "A Singular introduction to commutative algebra").

Problem 3: Let A be a ring and M a module over A represented by $A^m \xrightarrow{\varphi} A^n \rightarrow M \rightarrow 0$. Given the canonical bases on A^n and A^m , let S be the matrix representing φ , and let $F_0^A(M)$ be the ideal in A generated by all $n \times n$ -minors of S .

Prove that $F_0^A(M) \subseteq \text{Ann}_A(M)$ with $\sqrt{F_0^A(M)} = \sqrt{\text{Ann}_A(M)}$. More precisely, if M can be generated by n elements, then show that $\text{Ann}_A(M)^n \subseteq F_0^A(M)$. Finally, conclude that $\text{Supp}(M) = V(F_0^A(M))$.

NOTE: You may use the fact that the definition of $F_0^A(M)$ neither depends on the choice of basis on A^m , A^n , nor on choice of the representation φ . If you are interested, independence on the choice of basis is easily proven. To prove the independence on the choice of representation, restrict yourself to the case in which A is local with maximal ideal \mathfrak{m} , and that the image and kernel of your representations lie in the product of \mathfrak{m} and the respective module. Show that in this case is basically a base change. Given a general representation, try to construct another representation satisfying the condition of our restrictions

Problem 4: Change your SINGULAR procedure computing a Gröbner basis in such a way that

- the output is sorted in ascending order with respect to the leading monomials.
- it takes an optional parameter such that if this optional parameter is the string "reduced", the procedure returns a reduced Gröbner basis, and if this optional parameter is missing, the procedure just returns some standard basis as before.