TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

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Computer Algebra

Winter Semester 2014 - Problem Set 5

Due December 4, 2014, 10:00

Problem 1: (*Computing ideal quotients*) Let $>$ be a monomial ordering on $\mathrm{Mon}(x_1, \ldots, x_n)$, let $\mathfrak{a}, \mathfrak{b} \trianglelefteq K[x_1, \ldots, x_n]_>$ be two ideals in the localized polynomial ring with $\mathfrak{a} = \langle g_1, \ldots, g_r \rangle$, $\mathfrak{b} = \langle h_1, \ldots, h_s \rangle$, $g_i, h_j \in K[x_1, \ldots, x_n]$. Define $h := h_1 + t \cdot h_2 + \ldots + t^{r-1} h_r \in K[t, x_1, \ldots, x_n]$. Prove that

$$\mathfrak{a} : \mathfrak{b} = \langle (\langle g_1, \ldots, g_r \rangle_{K[t,x_1,\ldots,x_n]} : h) \cap K[x_1, \ldots, x_n] \rangle_{K[x_1,\ldots,x_n]_>}.$$

Problem 2: (*Zariski closure of the image*) Consider $\varphi : \mathbb{Q}^2 \to \mathbb{Q}^4$, $(s, t) \mapsto (s^4, s^3 t, s t^3, t^4)$. Compute the Zariski closure of the image, $\overline{\varphi(\mathbb{Q}^2)}$, and decide whether $\varphi(\mathbb{Q}^2)$ coincides with its closure.

Problem 3: Let $>$ be a monomial ordering on $\mathrm{Mon}(x_1, \ldots, x_n)$, and let $I \subseteq K[x_1, \ldots, x_n]$ be an ideal with primary decomposition $I = \bigcap_{i=1}^{t} Q_i$. Show that

$$I \cdot R \cap K[x_1, \ldots, x_n] = \bigcap_{i: Q_i R \neq R} Q_i,$$

where $R := K[x_1, \ldots, x_n]_>$.

Problem 4: Change your SINGULAR procedure computing a Gröbner basis in such a way that

1. the pair set P is sorted in ascending order w.r.t. $\mathrm{lcm}(\mathrm{LM}_>(f_1), \mathrm{LM}_>(f_2))$, $f_1, f_2 \in P$.
2. it takes an optional paramter such that if this optional parameter is the string "minimal", the procedure returns a minimal Gröbner basis, and if this optional parameter is missing, the procedure just returns some standard basis as before.

HINT: If you add `list #` at the end of the input of your procedure, then your procedure allows for any number of optional parameters. You can test with `size(#)` the number of optional parameters the user has provided, while `#[i]` gives you the i-th optional parameter.