Technische Universität Kaiserslautern

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Algebraic Geometry

Summer Semester 2015 - Problem Set 3

Due May 8, 2015, 11:00 am

In all exercises, the ground field k is assumed to be algebraically closed.

Problem 1.

(a) Find the irreducible components of the affine variety $V(x_1 - x_2x_3, x_1x_3 - x_2^2) \subset \mathbb{A}^3$.

(b) Let X be the set of 2×3 matrices with rank at most 1, considered as a subset of $\mathrm{Mat}(2,3) = \mathbb{A}^6$. Show that X is an affine variety. Decide whether X is irreducible and compute its dimension.

Problem 2. Let X, Y topological spaces and let $f : X \to Y$ be a continuous map. Show the following statements:

(a) A subset $A \subset X$ is irreducible if and only if $\bar{A}$ is irreducible.

(b) If X is irreducible then $f(X)$ is irreducible.

Problem 3. Let $X \subset \mathbb{A}^n$ and $Y \subset \mathbb{A}^m$ be irreducible affine varieties. Show that $X \times Y$ is an irreducible affine variety.

Hint: Consider the coordinate ring of $X \times Y$.

Problem 4. Let X be a topological space. Prove:

(a) If $\{U_i : i \in I\}$ is an open cover of X then $\dim(X) = \sup\{\dim(U_i) : i \in I\}$.

(b) If X is an irreducible affine variety and $U \subset X$ a non-empty open subset then $\dim(X) = \dim(U)$. Does this statement hold more generally for any irreducible topological space X?