Technische Universität Kaiserslautern

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Computer Algebra

Winter Semester 2015 - Problem Set 4

Due November 26, 2015, 10:00

Problem 1:

(a) Let $>$ be any monomial ordering, $R = K[x_1, \ldots, x_n]_>$, $I \subset R$ an ideal. Show that if I has a reduced standard basis, then it is unique.

(b) Show that Remark 1.7.2 in the SINGULAR book is not correct.

Problem 2:

(a) Show by example that reduced normal forms with respect to non-global orderings do in general not exist.

(b) Let $>$ be the ordering `ds`. Compute a standard representation of x_1 with respect to $\{x_1 - x_2, x_2 - x_1^2\}$ in $K[x_1, x_2]_>$.

Problem 3: (*Product Criterion*) Let $>$ be a global monomial ordering on $\text{Mon}(x_1, \ldots, x_n)$. Let $f, g \in K[x_1, \ldots, x_n]$ be polynomials such that $\text{lcm}(\text{LM}_>(f), \text{LM}_>(g)) = \text{LM}_>(f) \cdot \text{LM}_>(g)$. Prove that

$$\text{NF}(\text{spoly}(f, g) \mid \{f, g\}) = 0.$$

Hint: Assume that $\text{LC}_>(f) = \text{LC}_>(g) = 1$ and claim that $\text{spoly}(f, g) = -\,\text{tail}(g) \cdot f + \text{tail}(f) \cdot g$ is a standard representation.

Problem 4: Write a SINGULAR procedure to compute the reduced normal form of a given polynomial $f \in K[x_1, \ldots, x_n]$ with respect to a given finite list of polynomials $G \subseteq K[x_1, \ldots, x_n]$ and a global monomial ordering $>$ without the use of the commands `reduce` and `NF`.