Technische Universität Kaiserslautern

Prof. Dr. Mathias Schulze
Dipl.-Math. Cornelia Rottner
Fachbereich Mathematik

Computer Algebra

Winter Semester 2015 - Problem Set 8

Due January 7, 2016, 10:00

Problem 1: Let $>$ be a monomial ordering on $\text{Mon}(x_1, \ldots, x_n)$, $R = K[x_1, \ldots, x_n]_>$ and $S = R/I$, where $I \subseteq R$ is an ideal. Let M be a finitely presented S-module with presentation $S^q \xrightarrow{\varphi} S^p \to M \to 0$. Consider the submodules $N_1 = \langle g_1, \ldots, g_r \rangle$ and $N_2 = \langle h_1, \ldots, h_s \rangle \subseteq M$ with $f_i, h_j \in M$. Give an algorithm to compute the intersection $N_1 \cap N_2$.

Problem 2: Give an algorithm to obtain a minimal resolution in the case of local rings, i.e. a ring defined by a local monomial ordering on a polynomial ring, from Schreyer's resolution (see Algorithm 2.5.16 in "A Singular introduction to commutative algebra").

Problem 3: Let $R = \mathbb{Q}[x, y, z]/\langle x^2 + y^2 + z^2 \rangle$, $M = R^3/\langle (x, xy, xz) \rangle$ and let $N = R^2/\langle (1, y) \rangle$. Moreover, let $\varphi : M \to N$ be the R-module homomorphism given by the matrix

$$A = \begin{pmatrix} x^2 + 1 & y & z \\ yz & 1 & -y \end{pmatrix}$$

Using Singular,

(a) compute $\text{Ker}(\varphi)$ without using the command `modulo`,

(b) compute $\text{Im}(\varphi) \cap \langle \overline{(x^2, y^2)} \rangle$ without using the command `intersect`,

(c) compute $\text{Ann}_R(\text{Im}(\varphi))$.