TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

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Computer Algebra

Winter Semester 2015/16 - Problem Set 9

Due January 14, 2016, 10:00

Problem 1: Let A be a ring and M a module over A represented by $A^m \xrightarrow{\varphi} A^n \to M \to 0$. Given the canonical bases on A^n and A^m, let S be the matrix representing φ, and let $F_0^A(M)$ be the ideal in A generated by all $n \times n$-minors of S.
Prove that $F_0^A(M) \subseteq \operatorname{Ann}_A(M)$ with $\sqrt{F_0^A(M)} = \sqrt{\operatorname{Ann}_A(M)}$. More precisely, if M can be generated by n elements, then show that $\operatorname{Ann}_A(M)^n \subseteq F_0^A(M)$. Finally, conclude that $\operatorname{Supp}(M) = V(F_0^A(M))$.

NOTE: You may use the fact that the definition of $F_0^A(M)$ neither depends on the choice of basis on basis on A^m, A^n, nor on choice of the representation φ. If you are interested, independance on the choice of basis is easily proven. To prove the independence on the choice of representation, restrict yourself to the case in which A is local with maximal ideal $\mathfrak{m}$, and that the image and kernel of your representations lie in the product of $\mathfrak{m}$ and the respective module. Show that in this case is basically a base change. Given a general representation, try to construct another representation satisfying the condition of our restriction which has the same minors.

Problem 2: Let A be a ring. Prove that $\left(\bigcup_{\mathfrak{p}\in \operatorname{Ass}(A)} \mathfrak{p}\right) \setminus \{0\}$ is the set of zerodivisors of A. Moreover, if A is reduced, show that then $\left(\bigcup_{\mathfrak{p}\in \operatorname{Ass}(A) \text{ minimal}} \mathfrak{p}\right) \setminus \{0\}$ is already the set of zero-divisors of A.

Problem 3: Let $\psi : R \to S$ be a finite ring map. Find $k \in \mathbb{N}_{\geq 0}$ and $I \trianglelefteq R[t_1, \dots, t_k]$ such that

$$S \cong R[t_1, \dots, t_k]/I.$$