TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

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Computer Algebra

Summer Term 2019 - Sheet 2

Due Date: Thursday, May 02, 2019, 10:00 am

Exercise 5. Let $R = K[[x_1, \dots, x_n]]$ be a power series ring over a field K and $<$ a monomial ordering on $\mathrm{Mon}_n := \{x^\alpha \mid \alpha \in \mathbb{N}^n\}$. Similarly to the case of polynomial rings, one would like to define the leading monomial $\mathrm{LM}(f)$ of any power series $0 \neq f = \sum_\alpha a_\alpha x^\alpha \in R$ by

$$\mathrm{LM}(f) := \max\{x^\alpha \mid a_\alpha \neq 0\}.$$

When is this definition well-defined? Prove your claim!

Exercise 6. Let $n > 1$ and let $w_1, \dots, w_n \in \mathbb{R}$ be linearly independent over $\mathbb{Q}$. Define $>$ on Mon_n by setting $x^\alpha < x^\beta$ if $\sum_{i=1}^n w_i \alpha_i < \sum_{i=1}^n w_i \beta_i$.

(a) Prove that $>$ is a monomial ordering.

(b) Show that there is no matrix $A \in \mathrm{GL}(n, \mathbb{Q})$ defining this ordering.

Exercise 7.

(a) Consider a matrix ordering $>_A$ on Mon_n for some matrix $A \in \mathrm{GL}(n, \mathbb{Q})$. Let $M \subseteq \mathrm{Mon}_n$ be a finite set. Determine a weight vector $w \in \mathbb{Z}^n$ which induces $>_A$ on M.
Hint: Use the fact that $x^\alpha >_A x^\beta$ if and only if $A\alpha >_{lex} A\beta$ and Example 1.2.12.

(b) Determine integer weight vectors which induce `dp`, respectively `ds`, on $M := \{x_1^i x_2^j x_3^k \mid 1 \leq i, j, k \leq 5\} \subseteq \mathrm{Mon}_3$.