TECHNISCHE UNIVERSITÄT KAISERSLAUTERN

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Computer Algebra

Summer Term 2019 - Sheet 7

Due Date: Thursday, June 6, 2019, 10:00 am

Exercise 22. Show that the ring $\mathbb{Q}[s^4, s^3t, st^3, t^4]$ is isomorphic to $\mathbb{Q}[x_1, x_2, x_3, x_4]/I$, where

$$I = \langle x_2x_3 - x_1x_4, x_3^3 - x_2x_4^2, x_2 - x_1^2x_3, x_1x_3^2 - x_2^2x_4\rangle.$$

Exercise 23. Consider the following system of equations over the complex numbers:

$$x + \frac{y}{z} = 2 \tag{1}$$

$$y + \frac{z}{x} = 2 \tag{2}$$

$$z + \frac{x}{y} = 2 \tag{3}$$

Show that for any solution $(x_0, y_0, z_0) \in \mathbb{C}^3$ it holds that either $x_0 + y_0 + z_0 = 3$ or $x_0 + y_0 + z_0 = 7$.

Exercise 24. Let $r \in \mathbb{N}_{\geq 1}$ and $I \subseteq K[x_1, \ldots, x_n]^r$. We fix a global monomial ordering $>$.

(a) State an algorithm to compute a reduced normal form of an element $f \in K[x_1, \ldots, x_n]^r$ with respect to a finite set $G \subseteq K[x_1, \ldots, x_n]^r \setminus \{0\}$ and prove its correctness.

(b) Show that

$$K[x_1, \ldots, x_n]^r = I \oplus \left(\bigoplus_{m \notin L(I)} K \cdot m \right).$$

Exercise 25. Write a SINGULAR procedure `redBuchberger(ideal I)` to compute the reduced Gröbner basis of a given ideal $I \trianglelefteq K[x_1, \ldots, x_n]$ with respect to a global monomial ordering $>$ without the use of the commands `std` and `groebner`.