

# Irregular primes to two billion

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  - ▶ 1930, 1937, 1954, 1955, 1963, 1964, 1970, 1973, 1975, 1976, 1978, 1987, 1991, 1992, 1993, 1996 (various)
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- Accepted by Math Comp this morning 😊