A use of Galois groups in Arithmetic Dynamics

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For any one-variable polynomial F over a field, define the *factorization type* of F, denoted $\varphi(F)$, to be the multiset consisting of the degrees of the irreducible factors of F.

Let $f \in \mathbb{Q}[T][X]$ be a separable polynomial with leading coefficient $\ell(T)$ and discriminant $\Delta(T)$. Let G be the Galois group of f.

Theorem

Let $M_1, \ldots, M_r$ be representatives of all the conjugacy classes of maximal subgroups of G. For $i = 1, \ldots, r$ let F_i be the fixed field of M_i and let $p_i(T, X)$ be a monic irreducible polynomial in $\mathbb{Q}[T][X]$ such that $F_i/\mathbb{Q}(T)$ is generated by a root of $p_i(T, X)$. Suppose that $t \in \mathbb{Q}$ satisfies

$$\Delta(t) \cdot \ell(t) \cdot \prod_{i=1}^{r} \operatorname{disc} p_i(t, X) \neq 0.$$

If either $\varphi(f_t) \neq \varphi(f)$ or $G_t \ncong G$, then there is an index i such that the polynomial $p_i(t, X)$ has a root in k.