Real multiplication through explicit correspondences

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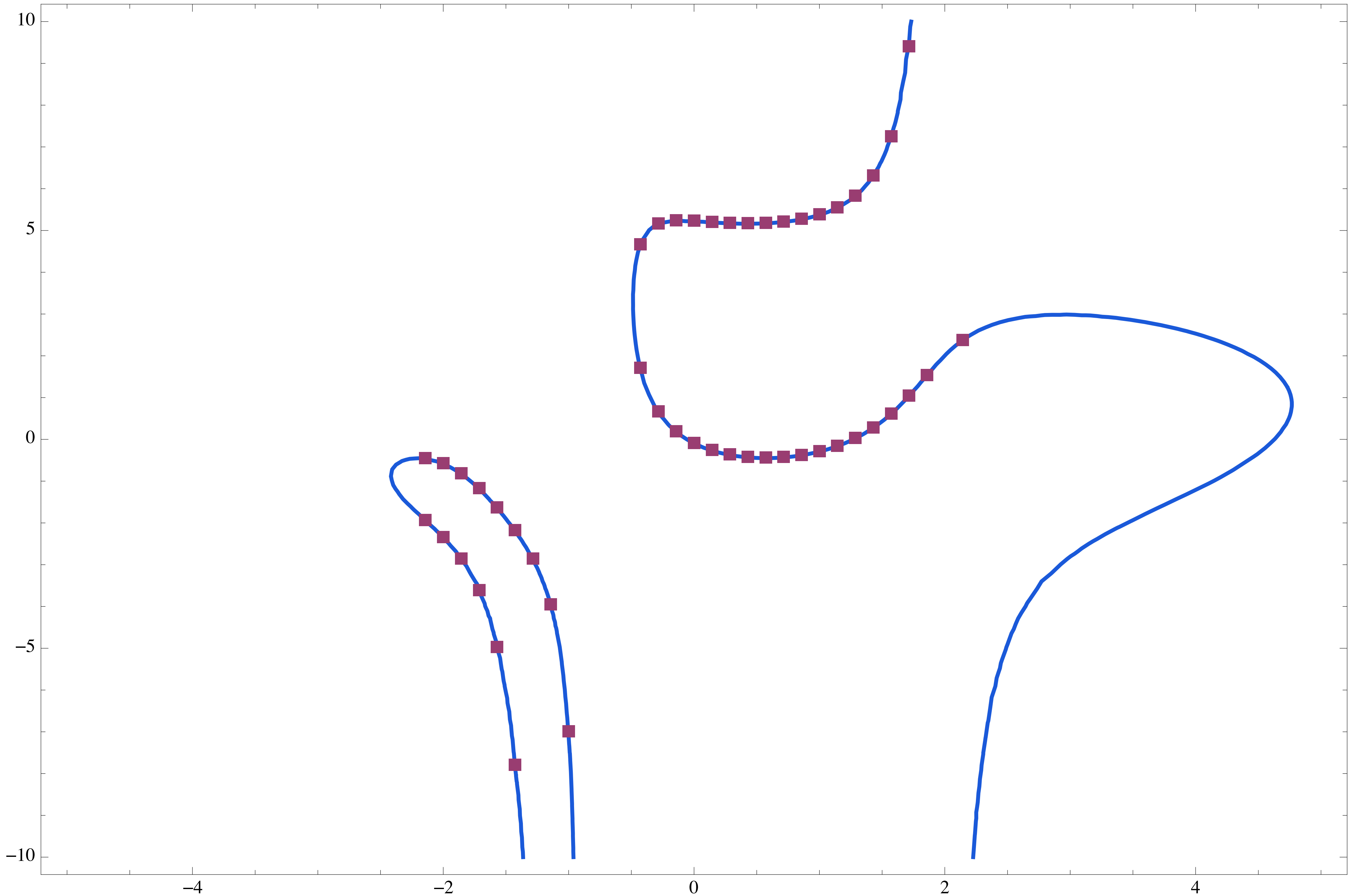
(joint with A. Kumar)



Sample Theorem

Theorem (Kumar-M.). *Let C be the curve $u^2 = t^5 - 2t^4 - 12t^3 - 8t^2 + 52t + 24$ and let $\phi : Z \to C$ be the degree two branched cover defined by*

$$16(t-2)(t+1)^2x^2 - (3t^4 + 16t^3 + 12t^2 - 192t - 164)x \\ +(9t^5 - 12t^4 - 140t^3 - 48t^2 + 276t + 16) = 0.$$

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$$y = -\frac{11t^4 - 24t^3 + 12t^2 - 112t - 132}{64(t-2)^2(t+1)^3} xu\sqrt{3} \\ -\frac{15t^5 - 28t^4 - 36t^3 + 288t^2 - 52t - 144}{64(t-2)^2(t+1)^3} u\sqrt{3}$$

is a square root of $f(x) = x^5 - 2x^4 - 12x^3 - 8x^2 + 52x + 24$, $\psi(t, u, x) = (x, y)$.

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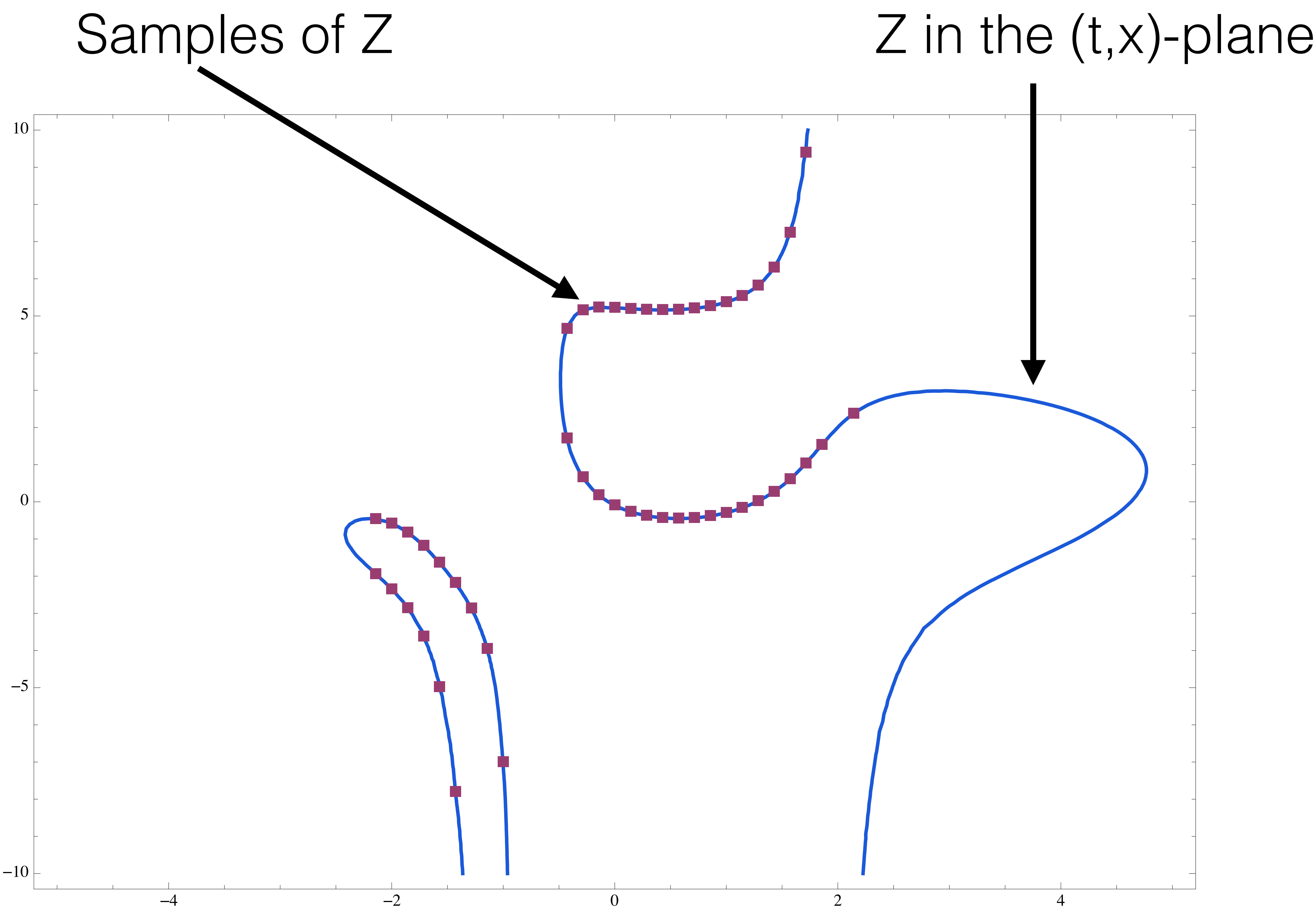
- Use [van Wamelen, ’00] to numerically sample the divisor Z in C x C
- Interpolate to find *exact* equations for Z



Samples of Z
Z in the (t,x)-plane
10
5
0
−5
−10
−4
−2
0
2
4

Correspondences in families

Theorem (Kumar-M.). *For* $(p,q) \in \mathbb{C}^2$, *let* $C(p,q)$ *be the curve defined by the equation*

$$u^2 = t^6 + 2pt^5 + 10qt^3 + 10q^2t - 5(p-1)q^2$$

and let $\phi : Z(p,q) \to C(p,q)$ *be degree two branched cover defined by*

$$\begin{aligned}(2t-p)\big(4t+(3+\alpha)p\big)x^2 &+ \big(2(-\alpha-1)u \\ &+ \alpha(2t^3 - 2pt^2 + p^2t + 2q) - (6t^3 - 6pt^2 - p^2t - 10q)\big)x \\ &- 2\big((1-\alpha)t - p\big)u + \alpha(2t^4 - p^2t^2 + 6qt - 4pq) \\ &- (2t^4 - 2pt^3 + 3p^2t^2 - 10qt + 10pq) = 0\end{aligned}$$

where $\alpha = \sqrt{5}$. *For generic* $(p,q) \in \mathbb{C}^2$, *the curve* $Z(p,q)$ *is of genus* 8 *and admits a holomorphic map* ψ *to* $C(p,q)$ *of degree* 3. *The endomorphism* $T = \psi_* \circ \phi^*$ *of* $\mathrm{Jac}(C(p,q))$ *satisfies* $T^2 - T - 1 = 0$.