Computing Jacobi forms

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Motivation

Computing central values (Gross–Kohnen–Zagier)

Let $f \in S^-_{2k-2}(m)$ be a newform, and let $S^-_{k,m} \ni \phi \longleftrightarrow f$. Then

$$C_{k,m} \, |\Delta|^{k-3/2} \cdot \frac{L(f \otimes \Delta, k-1)}{\langle f, f \rangle} = \frac{|c_\phi(\Delta, r)|^2}{\langle \phi, \phi \rangle}$$

for every fundamental $\Delta < 0$ with $(\Delta, m) = 1$, $\Delta \equiv r^2 \bmod 4m$.

Verifying the paramodular conjecture (Brumer–Kramer)

Let $A = \mathrm{Jac}(C)$ where

$$C : \quad y^2 + y = x^5 + 5x^4 + 8x^3 + 6x^2 + 2x,$$

and let $S_2(K(277)) \ni F = \sum_{m \geq 1} \phi_{277m} \, q'^m$ be the non-lift. Then

$$L(A, s) \stackrel{?}{=} L(F, s).$$

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Jacobi forms basics

Fix $\varepsilon \in \{\pm 1\}$. A Jacobi form $S^{\varepsilon}_{k,m} \ni \phi : \mathbb{H} \times \mathbb{C} \to \mathbb{C}$ has a Fourier expansion

$$\phi(\tau, z) = \sum_{\substack{\Delta, r \in \mathbb{Z},\ \varepsilon\Delta > 0 \\ \Delta \equiv r^2 \bmod 4m}} c_\phi(\Delta, r)\, e^{2\pi i \left(\frac{r^2 - \Delta}{4m} u + \frac{r^2 + |\Delta|}{4m} iv + rz \right)}.$$

with $c_\phi(\Delta, r)$ depending only on $r \bmod 2m$.

In the space $S^{\varepsilon}_{k,m}$ we have:

- A skew-linear involution $\jmath : \phi(\tau, z) \mapsto \overline{\phi(-\overline{\tau}, -\overline{z})}$.
- An inner product $\langle\, , \rangle$.
- Action of self-adjoint Hecke operators T_p.
- Old and newforms.

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The Skoruppa–Zagier lift

Fix an *admissible pair* (Δ_0, r_0), i.e.: Δ_0 is fundamental with $\mathrm{sgn}(\Delta_0) = \varepsilon$, and $\Delta_0 \equiv r_0^2 \bmod 4m$.

Theorem (Skoruppa–Zagier)

The map $\mathcal{S}_{\Delta_0,r_0} : S^{\varepsilon}_{k,m} \to S^{\varepsilon}_{2k-2}(m)$ *given by*

$$\mathcal{S}_{\Delta_0,r_0}(\phi) = \sum_{n\geq 1} \left(\sum_{d|n} d^{k-2} \left(\frac{\Delta_0}{d}\right) c_\phi \left(\frac{n^2}{d^2}\Delta_0, \frac{n}{d} r_0\right) \right) q^n$$

is Hecke linear. Furthermore, there is an isomorphism

$$\sum_i \alpha_i \, \mathcal{S}_{\Delta_i,r_i} : S^{\varepsilon,new}_{k,m} \xrightarrow{\simeq} S^{\varepsilon,new}_{2k-2}(m).$$

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S-Z's dual map (on modular symbols)

Let $\mathbb{M}_{2k-2}(m) = \left\{ \sum_i n_i\{\alpha_i, \beta_i\} \otimes P_i \right\}$.

Let $\sigma \mapsto \sigma^\varepsilon$ denote the projection $\mathbb{M}_{2k-2}(m) \to \mathbb{M}^\varepsilon_{2k-2}(m)$.

Definition

$\Sigma^*_{\Delta_0,r_0} : \mathbb{M}_{2k-2}(m) \to S^\varepsilon_{k,m}$ is the (Hecke linear) map satisfying

$$\left\langle \phi, \jmath\Sigma^*_{\Delta_0,r_0}(\sigma) \right\rangle = \langle \mathcal{S}_{\Delta_0,r_0}(\phi), \sigma^\varepsilon \rangle \qquad \forall\, \phi \in S^\varepsilon_{k,m}.$$

Let $\mathcal{Q}_m(\Delta, r) = \{Q = [ma, b, c] : \operatorname{disc} Q = \Delta,\ b \equiv r \mod 2m\}$.

Theorem (Skoruppa, Ryan–S.–Skoruppa–Tornaría)

Let $\sigma \in \mathbb{S}_{2k-2}(m)$, and let $\phi = \Sigma^*_{\Delta_0,r_0}(\sigma)$. Then if $\Delta\Delta_0 \neq \square$

$$c_\phi(\Delta, r) = \sum_{Q \in \mathcal{Q}_m(\Delta\Delta_0, rr_0)} \chi_{m,\Delta_0}(Q)\, C_Q \cdot \sigma.$$

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Implementation of the formula

Goal $(0 < |\Delta| < \Delta_{\max})$

$$\sum_{Q \in \mathcal{Q}_m(\Delta\Delta_0, rr_0)} \chi_{m,\Delta_0}(Q)\, C_Q \cdot \sigma = ???$$

- Write $\sigma = \sum_i n_i \{\infty, s_i\} \otimes P_i$. If $\operatorname{disc} Q \neq \square$, then

$$C_Q \cdot \sigma = \operatorname{sgn} Q(\infty) \sum_{Q(\infty)\, Q(s_i) < 0} n_i\, [P_i \mid Q^{k-2}].$$

- Let $Q = [ma, b, c]$ with $Q(\infty) > 0$. Assume that $0 < \operatorname{disc} Q < D_{\max}$. Let $s = p/q \in \mathbb{Q}$ with $Q(s) < 0$. Then

$$\begin{aligned} 0 < ma &\leq \tfrac{D_{\max} q^2}{4}, \\ \left\lfloor -2mas - \sqrt{D_{\max}} \right\rfloor < \quad b \quad &< \left\lceil -2mas + \sqrt{D_{\max}} \right\rceil, \\ \left\lceil \tfrac{b^2 - D_{\max}}{4ma} \right\rceil \leq \quad c \quad &< \left\lceil \tfrac{b^2 - (b+2mas)^2}{4ma} \right\rceil. \end{aligned}$$

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Example: $\sigma \longleftrightarrow$ '37.a1'

$$\sigma = \{\infty, -\tfrac{1}{23}\} - \{\infty, -\tfrac{1}{32}\} + \{\infty, -\tfrac{1}{34}\} - \{\infty, 0\} \in \mathbb{S}_2^-(37).$$

Δ	r	$c_{-4,12}(\Delta, r)$	$c_{-3,21}(\Delta, r)$
-3	21	1	NA
-4	12	NA	1
-7	17	-1	-1
-11	27	1	1
-12	32	-1	NA
-16	24	NA	-2
-27	11	-3	NA
-28	34	3	3
-36	36	NA	-2
-40	16	2	2
-44	20	-1	-1
-47	29	-1	-1

Timing (with $\Delta_{\max} = 10.000$): 38,5 s.

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Example: $\sigma \longleftrightarrow$ '389.a1'

$$\sigma = -2\{\infty, 0\} + \{\infty, -\tfrac{1}{261}\} + 2\{\infty, -\tfrac{1}{276}\} + \cdots \in \mathbb{S}_2^+(389).$$

Δ	r	$c(\Delta, r)$
1	1	0
4	2	0
5	303	1
9	3	0
13	205	-1
16	4	0
17	79	-1
20	172	1
24	56	1
25	5	0
28	240	-1
36	6	0
41	279	-1
44	40	-1

Timing (with $\Delta_{\max} = 10.000$): 37,2 s.

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Thanks!

Danke

Gracias